

科目：工程數學【機電系碩士班】✓

I.

1. (15%) Solve the general solution of the following non-homogeneous differential equation

$$y'' + y = \cos x + \sec x$$

2. (10%) Experiments show that in a temperature field  $T = x^3 - 3xy^2$ , heat flows in the direction of maximum decrease of temperature  $T$ . Find this direction in general and at a given point  $P: (\sqrt{8}, \sqrt{2})$ .

II.

1. (7%) For  $f(t) = 3 + \sqrt{3}\cos 2t + \sin 2t + \sin 3t - \cos(5t + \pi/3)/2$ , find the compact Fourier series representation for  $f(t)$ .
2. (10%) Determine which of the following functions are periodic. For periodic functions, find the period.
- $3\sin t + 2\sin 3t$
  - $2 + 5\sin 4t + 4\cos 7t$
  - $2\sin 3t + 7\cos \pi t$
  - $7\cos \pi t + 5\sin 2\pi t$
  - $\sin(5t/2) + 3\cos(6t/5) + 3\sin(t/7 + 30^\circ)$
3. (8%) Determine  $f(t)$  from  $f(t) = -1 + f(t) * e^{-3t}$ . Note that the term  $f(t) * e^{-3t}$  represents the convolution integral of  $f(t)$  and  $e^{-3t}$ .

III.

1. With a matrix  $A$  and a vector  $b$  given:

$$A = \begin{bmatrix} 1 & 3 & 5 & 4 \\ -2 & 2 & 6 & 2 \\ 3 & 1 & -1 & 3 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

- Find the rank of  $A$ . (3%)
- Find the homogeneous solution to  $Ax = 0$ . (2%)
- Solve  $Ax = b$  (5%) and explain the relation between  $x$  and the homogeneous solution of  $Ax = 0$  (3%).

2. With a matrix  $A$  given:

$$A = \begin{bmatrix} 0 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 0 \end{bmatrix}$$

Compute the eigenvalues and eigenvectors of  $A$  (12%).

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IV.

1. (14%) Evaluate the following integrals (counterclockwise) using the methods

discussed in Complex Analysis. (a)  $\oint_C \frac{z+1}{z^4 - 2z^3}$ ,  $C: |z|=3$ ; (b)  $\oint_C \frac{z^5}{1-z^3}$ ,  $C:$ 

$$|z|=3.$$

2. (11%) Solve the homogeneous Euler-Cauchy equation

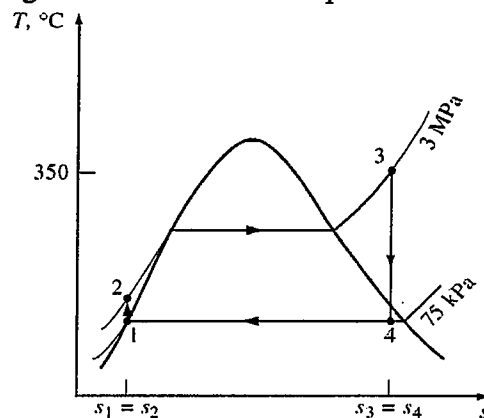
$$x^3 y''' + x^2 y'' - 2xy' + 2y = 0.$$

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請在答案卷上作答

**Part I: Thermodynamics (65%)**

- 5%. What are the meanings of heat and work? Discuss the similarities and differences between heat and work?
- 5%. A substance whose Joule-Thomson coefficient is negative is throttled to a lower pressure. During this process, (select the correct statement)
  - the temperature of the substance will increase.
  - the temperature of the substance will decrease.
  - the entropy of the substance will remain constant.
  - the entropy of the substance will decrease.
  - the enthalpy of the substance will decrease.
- 10%. If an open system is interactive with its surroundings through the three mechanisms mass, heat and work transfers. What are the possible mechanisms, which can cause the changes of the internal energy, the entropy, and the exergy of a control volume, respectively? Please discuss the change one by one.
- 15%. Derive relations for the specific enthalpy and entropy changes of a gas that obeys the Nobel-Abel equation of state  $p(v-b) = RT$ . Assume that in the range of interest  $c_p$  varies according to the relation  $c_p = c_1 + c_2 T$ , where  $c_1$  and  $c_2$  are constants.  
Note:  $dh = c_p dT + [v - T(\partial v/\partial T)_p] dp$ , and  $ds = (c_p/T)dT - (\partial v/\partial T)_p dp$
- 15%. Compare the vapor-compression refrigeration cycle with an absorption refrigeration cycle by drawing the schematics for the two cycles and explaining the differences.(10%) Individually explain how do you use the solar energy to drive these two refrigeration systems.(5%)
- 15%. Consider a steam power plant operating on the simple ideal Rankine cycle. Steam enters the turbine at 3 MPa and 350°C and is condensed in the condenser at a pressure of 75 kPa. Determine the thermal efficiency of this cycle. Also, determine the thermal efficiency of a Carnot cycle operating between the same temperature limits. (Tables are given in the next page.)



**Part II: Conduction and Radiation (35%)**

- 15%. A copper slab ( $k=372$  W/m K) is 3 mm thick. It is protected from corrosion by a 2-mm-thick layers of stainless steel ( $k=17$  W/m K) on both sides. The temperature is 400 °C on one side of this composite wall and 100 °C on the other. Find the temperature distribution in the copper slab and the heat conduction through the wall.
- 15%. A large thin concrete slab of thickness  $L$  is “setting.” Setting is an exothermic process that releases  $\dot{q}$  W/m<sup>3</sup>. The out side surfaces are kept at the ambient temperature, so  $T_w = T_\infty$ . Solve the temperature distribution in terms of  $\dot{q}$ ,  $L$  and  $T_\infty$ . What is the maximum internal temperature? (hint: Write down the appropriate steady heat conduction equation and solve the equation with the given boundary conditions.)
- 5%. Why glass is used as the “green house” material?

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Saturated water—Temperature table (Continued)

Temp., T °C	Sat. press., P <sub>sat</sub> kPa	Specific volume, m <sup>3</sup> /kg		Internal energy, kJ/kg			Enthalpy, kJ/kg			Entropy, kJ/kg · K		
		Sat. liquid, v <sub>f</sub>	Sat. vapor, v <sub>g</sub>	Sat. liquid, u <sub>f</sub>	Evap., u <sub>fg</sub>	Sat. vapor, u <sub>g</sub>	Sat. liquid, h <sub>f</sub>	Evap., h <sub>fg</sub>	Sat. vapor, h <sub>g</sub>	Sat. liquid, s <sub>f</sub>	Evap., s <sub>fg</sub>	Sat. vapor, s <sub>g</sub>
330	12,858	0.001560	0.012979	1505.7	993.5	2499.2	1525.8	1140.3	2666.0	3.5516	1.8906	5.4422
335	13,707	0.001597	0.011848	1537.5	945.5	2483.0	1559.4	1086.0	2645.4	3.6050	1.7857	5.3907
340	14,601	0.001638	0.010783	1570.7	893.8	2464.5	1594.6	1027.4	2622.0	3.6602	1.6756	5.3358
345	15,541	0.001685	0.009772	1605.5	837.7	2443.2	1631.7	963.4	2595.1	3.7179	1.5585	5.2765
350	16,529	0.001741	0.008806	1642.4	775.9	2418.3	1671.2	892.7	2563.9	3.7788	1.4326	5.2114

Saturated water—Pressure table (Continued)

Press., P kPa	Sat. temp., T <sub>sat</sub> °C	Specific volume, m <sup>3</sup> /kg		Internal energy, kJ/kg			Enthalpy, kJ/kg			Entropy, kJ/kg · K		
		Sat. liquid, v <sub>f</sub>	Sat. vapor, v <sub>g</sub>	Sat. liquid, u <sub>f</sub>	Evap., u <sub>fg</sub>	Sat. vapor, u <sub>g</sub>	Sat. liquid, h <sub>f</sub>	Evap., h <sub>fg</sub>	Sat. vapor, h <sub>g</sub>	Sat. liquid, s <sub>f</sub>	Evap., s <sub>fg</sub>	Sat. vapor, s <sub>g</sub>
75	91.76	0.001037	2.2172	384.36	2111.8	2496.1	384.44	2278.0	2662.4	1.2132	6.2426	7.4558
100	99.61	0.001043	1.6941	417.40	2088.2	2505.6	417.51	2257.5	2675.0	1.3028	6.0562	7.3589
101.325	99.97	0.001043	1.6734	418.95	2087.0	2506.0	419.06	2256.5	2675.6	1.3069	6.0476	7.3545
125	105.97	0.001048	1.3750	444.23	2068.8	2513.0	444.36	2240.6	2684.9	1.3741	5.9100	7.2841
150	111.35	0.001053	1.1594	466.97	2052.3	2519.2	467.13	2226.0	2693.1	1.4337	5.7894	7.2231
1750	205.72	0.001166	0.11344	876.12	1720.6	2596.7	878.16	1917.1	2795.2	2.3844	4.0033	6.3877
2000	212.38	0.001177	0.099587	906.12	1693.0	2599.1	908.47	1889.8	2798.3	2.4467	3.8923	6.3390
2250	218.41	0.001187	0.088717	933.54	1667.3	2600.9	936.21	1864.3	2800.5	2.5029	3.7926	6.2954
2500	223.95	0.001197	0.079952	958.87	1643.2	2602.1	961.87	1840.1	2801.9	2.5542	3.7016	6.2558
3000	233.85	0.001217	0.066667	1004.6	1598.5	2603.2	1008.3	1794.9	2803.2	2.6454	3.5402	6.1856

Superheated water (Continued)

T °C	v m <sup>3</sup> /kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m <sup>3</sup> /kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m <sup>3</sup> /kg	u kJ/kg	h kJ/kg	s kJ/kg · K
P = 2.50 MPa (223.95°C)				P = 3.00 MPa (233.85°C)				P = 3.50 MPa (242.56°C)				
Sat.	0.07995	2602.1	2801.9	6.2558	0.06667	2603.2	2803.2	6.1856	0.05706	2603.0	2802.7	6.1244
225	0.08026	2604.8	2805.5	6.2629								
250	0.08705	2663.3	2880.9	6.4107	0.07063	2644.7	2856.5	6.2893	0.05876	2624.0	2829.7	6.1764
300	0.09894	2762.2	3009.6	6.6459	0.08118	2750.8	2994.3	6.5412	0.06845	2738.8	2978.4	6.4484
350	0.10979	2852.5	3127.0	6.8424	0.09056	2844.4	3116.1	6.7450	0.07680	2836.0	3104.9	6.6601
400	0.12012	2939.8	3240.1	7.0170	0.09938	2933.6	3231.7	6.9235	0.08456	2927.2	3223.2	6.8428

科目：流體力學及熱對流【機電系碩士班甲組】 ✓

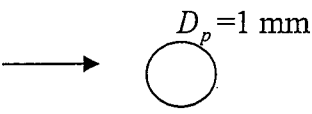
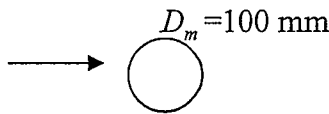
請在答案卷上作答

- 10%. Nomenclature
  - fully-developed flow
  - Reynolds stress
  - irrotational flow
  - hydraulic diameter
  - Newtonian fluid
- 5%. What is the major difference between Lagrangian approach and Eulerian approach? What type of the approach is commonly used in viscous flow problem? Why?
- 10%. The velocity field of a flow is given by  $u = -V_0 y / (x^2 + y^2)^{1/2}$  and  $v = V_0 x / (x^2 + y^2)^{1/2}$ , where  $V_0$  is constant. Where in the flow field is the speed equal to  $V_0$ ? Determine the equation of the streamlines and discuss the various characteristics of the flow.
- 5%. What are the main differences in drag force between flow past a streamline body and flow past a blunt body? Why?
- 5%. Explain the three types of flows: laminar, transitional, and turbulent flow. What are their characteristics, respectively?

- 12%. The equation of motion in the x-direction is

$$\rho g_x + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = \rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right)$$

- Write down the corresponding equation of motion in the y-direction.
  - What are the stress tensor,  $\sigma_{xx}$ ,  $\tau_{yx}$ , and  $\tau_{zx}$ , if the fluid flow is inviscid.
- 18%. A copepod is a water crustacean approximately 1 mm in diameter. We want to know the drag force on the copepod when it moves slowly in fresh water. A scale model 100 times larger is made and tested in glycerin at  $V = 30 \text{ cm/s}$ . The measured drag on the model is 1.3 N. Assume the temperature is  $20^\circ \text{C}$ , the properties are listed below

Water	Glycerin
	
$V_p = ?$	$V_m = 30 \text{ cm/s}$
$\rho_p = 998 \text{ kg/m}^3$	$\rho_m = 1263 \text{ kg/m}^3$
$\mu_p = 0.001 \text{ kg/m}\cdot\text{s}$	$\mu_m = 1.5 \text{ kg/m}\cdot\text{s}$
$F_p = ?$	$F_m (\text{measured}) = 1.3 \text{ N}$

We know the force on the sphere is a function of  $D$ ,  $V$ ,  $\rho$ , and  $\mu$ . i.e.  $F = f(D, V, \rho, \mu)$

- Non-dimensionalize the functional relationship.
- For similar conditions, what is the velocity  $V_p = ?$
- The drag of the actual copepod in water  $F_p = ?$

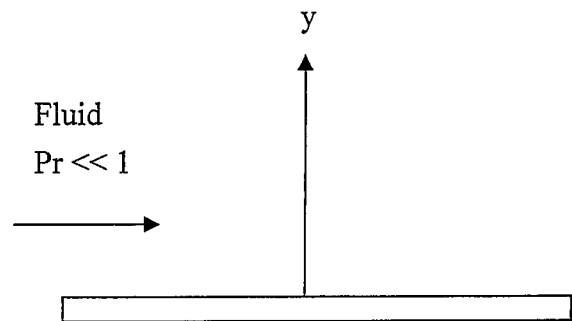
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8. (10%) Give the definition of Prandtl number and its physical meaning. Sketch the dimensionless velocity and temperature profiles in the boundary layer of a liquid metal flowing horizontally over a cold flat plate (The liquid metal has very small Prandtl number, say 0.01). Dimensionless velocity and temperature are respectively defined as

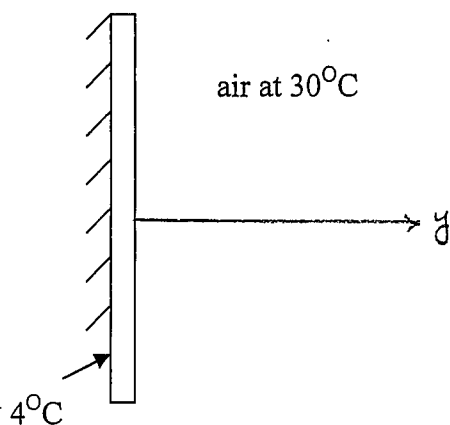
$$\bar{u}(y) = \frac{u(y)}{U_\infty}$$

$$\bar{T}(y) = \frac{T(y) - T_s}{T_\infty - T_s}$$

where  $U_\infty$  and  $T_\infty$  are free stream velocity and temperature.  $T_s$  is the surface temperature. (Be careful about the relative magnitude of the boundary layer thicknesses of  $u(y)$ , and  $T(y)$ )

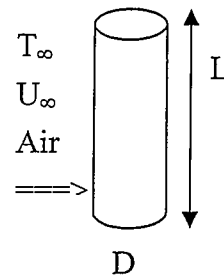


9. (5%) Draw velocity and temperature profiles in the boundary layer of a cool vertical flat plate (natural convection) in hot air.



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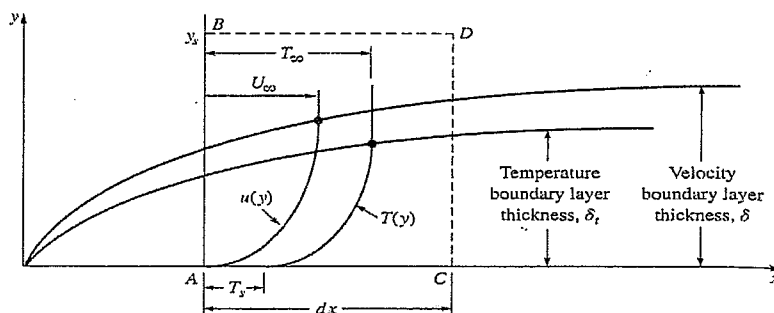
10 (10%) For a vertical cylinder in the air with kinematics viscosity  $\nu$ , thermal diffusivity  $\alpha$ , volumetric coefficient of thermal expansion  $\beta$ , the convective heat transfer coefficient averaged over the cylinder surface is  $hc$ . The length  $L$  is larger than the diameter  $D$  ( $D \ll L$ ). Gravitational acceleration is  $g$ . The surface temperature of the cylinder is  $T_s$ . ( $T_s > T_\infty$ )



- (a) In case that a horizontal air flow with velocity  $U_\infty$  approaches the cylinder and forced convection is dominant (compared to natural convection). What are the two major dimensionless parameters (grouped from the above dimensional quantities) that the Nusselt number of the forced convection depends?
- (b) In case that the free stream velocity is zero (i.e.  $U_\infty=0$ ), there is only natural convection. What are the two major dimensionless parameters (grouped from the above dimensional quantities) that the Nusselt number depends?
- (c) In the case of  $U_\infty=0$ , only natural convection occurs. Which of the following two cases would result better heat transfer rate (larger  $hc$ )? Explain your answer.  
 Case I: The cylinder is vertical.  
 Case II: The cylinder is horizontal.

11. (10%) Derive the integral energy equation of the laminar boundary layer for low-speed flow (Hint: consider the control volume shown in the following figure)

$$\frac{d}{dx} \int_0^{\delta_t} (T_\infty - T) u dy - \alpha \left( \frac{\partial T}{\partial y} \right)_{y=0} = 0$$



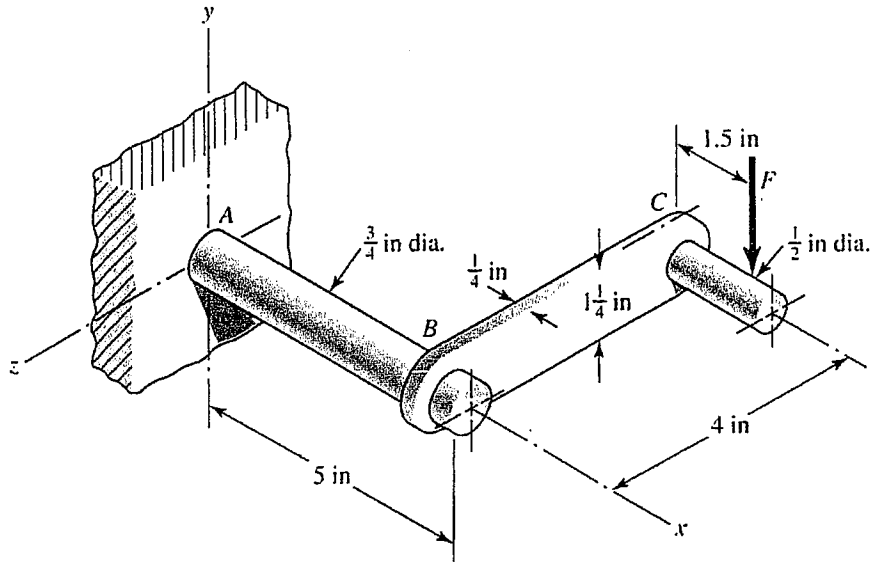
科目：材料力學【機電系碩士班乙組】 ✓

**第一部份選擇題 50%** (以下題目為單選題，所有答案均須寫於答案卷紙上。計分採倒扣制，每題答對得五分，不答得零分，回答但答錯則得負一分)

- (I) A hot-rolled steel specimen has a yield strength of  $S_{yt} = S_{yc} = 100 \text{ kpsi}$ . The Young's modulus and the Poisson's ratio of the steel are  $E = 30 \times 10^6 \text{ psi}$  and  $\nu = 0.3$ . Estimate the factor of safety for the principal stress states  $\sigma_1 = 25 \text{ kpsi}$ ,  $\sigma_2 = 25 \text{ kpsi}$  and  $\sigma_3 = -25 \text{ kpsi}$
- (1) ( ) By using the maximum normal stress theory, the factor of safety is  
 (A)  $n = 0.5$  (B)  $n = 1.0$   
 (C)  $n = 2.0$  (D)  $n = 4.0$   
 (E)  $n = \infty$
- (2) ( ) By using the maximum shear stress theory, the factor of safety is  
 (A)  $n = 0.5$  (B)  $n = 1.0$   
 (C)  $n = 2.0$  (D)  $n = 4.0$   
 (E)  $n = \infty$
- (3) ( ) By using the maximum distortion energy theory, the factor of safety is  
 (A)  $n = 0.5$  (B)  $n = 1.0$   
 (C)  $n = 2.0$  (D)  $n = 4.0$   
 (E)  $n = \infty$
- (II) A specimen of annealed Titanium having an initial diameter of  $d_o = 0.505 \text{ in}$  was tested in tension using a gauge length of  $\ell_o = 2.000 \text{ in}$ . A gauge length of  $\ell = 2.310 \text{ inch}$ , i.e. a cross section area  $A = 0.1735 \text{ in}^2$ , was measured under a tensile load of  $P = 14950 \text{ lbf}$ .
- (4) ( ) The engineering stress of the specimen at this load is (A) 74.6 kpsi (B) 86.2 kpsi  
 (C) 69.9 kpsi (D) 98.7 kpsi (E) None
- (5) ( ) The true stress of the specimen at this load is (A) 74.6 kpsi (B) 86.2 kpsi  
 (C) 69.9 kpsi (D) 98.7 kpsi (E) None
- (6) ( ) The engineering strain of the specimen at this load is (A) 0.1132 in/in (B) 0.1550 in/in  
 (C) 0.3057 in/in (D) 0.4132 in/in (E) None
- (7) ( ) The true strain of the specimen at this load is (A) 0.1441 in/in (B) 0.2548 in/in  
 (C) 0.3057 in/in (D) 0.4132 in/in (E) None
- (III) A crank is loaded by a force  $F = 300 \text{ lbf}$  that causes twisting and bending of a 3/4 inch diameter shaft fixed to a support at the origin of the reference system.



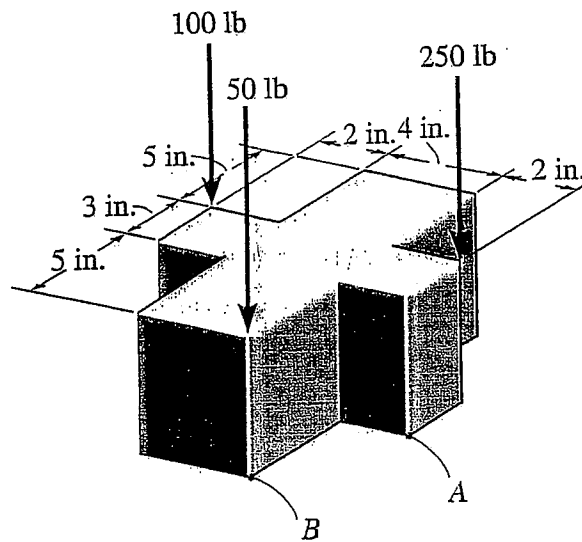
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- (8) ( ) The torsional stress at a stress element  $A$  is (A)  $\tau_{xz} = -14500 \text{ psi}$  (B)  $\tau_{xz} = -44500 \text{ psi}$   
 (C)  $\tau_{xz} = 14500 \text{ psi}$  (D)  $\tau_{xz} = 44500 \text{ psi}$  (E) None
- (9) ( ) The maximum principal stress at the point  $A$  is (A)  $\sigma_1 = 3.82 \text{ kpsi}$  (B)  $\sigma_1 = 47.1 \text{ kpsi}$   
 (C)  $\sigma_1 = 51.2 \text{ kpsi}$  (D)  $\sigma_1 = 93.87 \text{ kpsi}$  (E) None
- (10) ( ) The maximum shear stress at the point  $A$  is (A)  $\tau_{\max} = 2.56 \text{ kpsi}$  (B)  $\tau_{\max} = 4.76 \text{ kpsi}$   
 (C)  $\tau_{\max} = 11.2 \text{ kpsi}$  (D)  $\tau_{\max} = 27.7 \text{ kpsi}$  (E) None

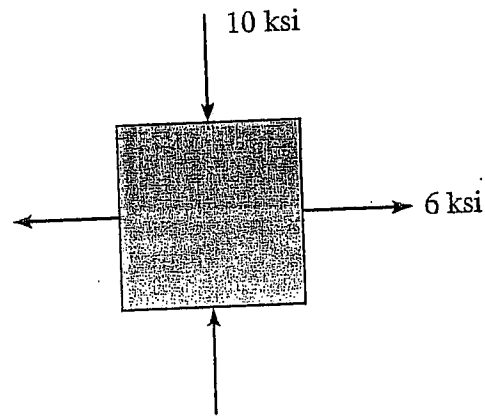
第二部份計算題 50%

- (I) The block is subjected to the three axial loads shown. Determine the normal stress developed at points  $A$  and  $B$ . neglect the weight of the block. (15%)

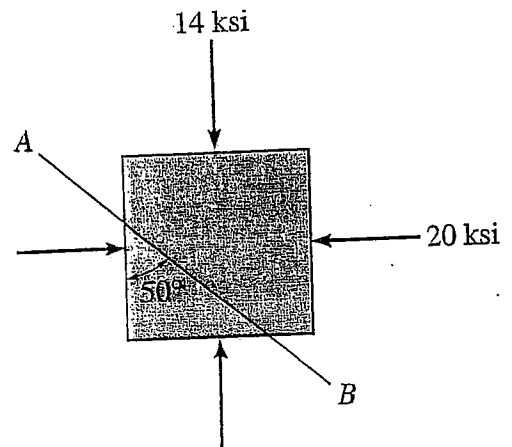


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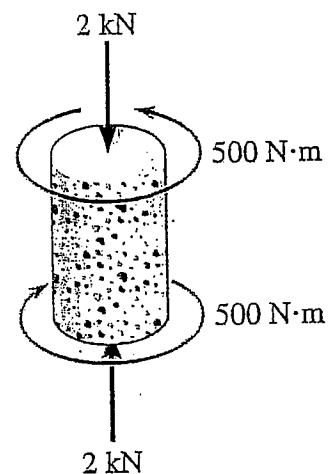
- (II) Determine the equivalent state of stress if an element is oriented  $40^\circ$  clockwise from the element shown. Use Mohr's circle. (10%)



- (III) The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane  $AB$ . (10%)



- (IV) The short concrete cylinder having a diameter of 50 mm is subjected to a torque of 500 Nm and an axial compressive force of 2 kN. Determine if it fails according to the maximum-normal-stress theory. The ultimate stress of the concrete is  $\sigma_{ult} = 28 \text{ MPa}$ . (15%)



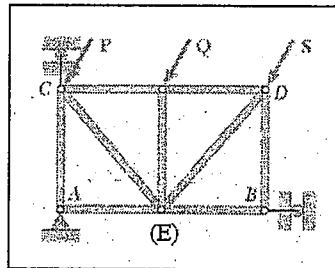
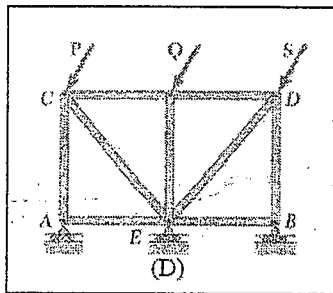
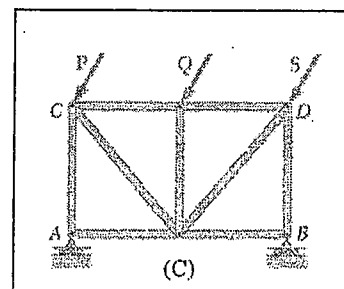
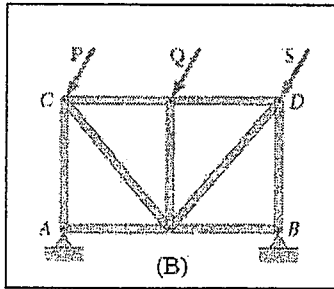
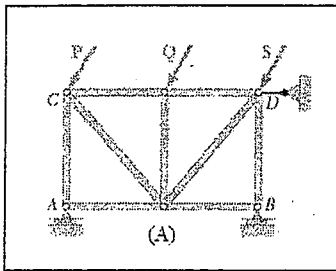
科目：應用力學【機電系碩士班乙組、丙組】

請在答案卷上作答！

Please choose the correct answers for problem 1 to problem 3

Please be noted that the correct answers for each problem may be more than one

1. Which systems are statically indeterminate systems? (5%)



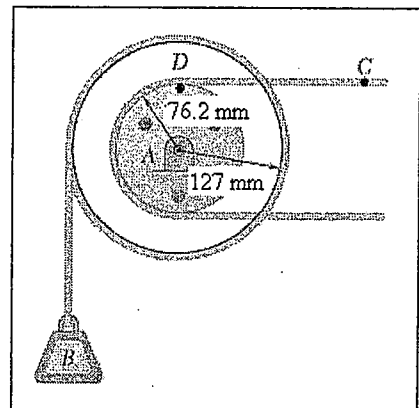
2. If there is no external forces act on the particles of a system, then which following statements are correct? (5%)

- (A) The linear momentum of the system is conserved.
- (B) The angular momentum of the system about a fixed point O is conserved.
- (C) The angular momentum of the system about the mass center G is conserved.
- (D) The velocity of the mass center of the system is constant.
- (E) The acceleration of each particle in the system is zero.
- (F) None of the previous statements is correct

3. Load B is connected to a double pulley by one of the two inextensible cables shown. The motion of the pulley is controlled by cable C, which has a constant acceleration of  $228 \text{ mm/s}^2$  and an initial velocity of  $304.8 \text{ mm/s}$ , both directed to the right.

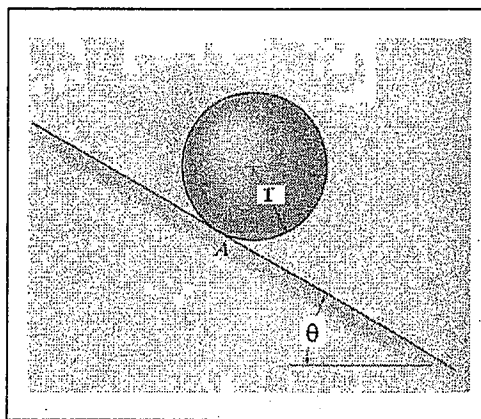
Which following statements are correct? (15%)

- (A) The initial angular velocity of the pulley is greater than  $5 \text{ rad/s}$ .
- (B) The initial angular acceleration of the pulley is less than  $2 \text{ rad/s}^2$ .
- (C) The velocity of the load B after 2 seconds is greater than  $1200 \text{ mm/s}$ .
- (D) The number of revolutions executed by the pulley in 2 seconds is less than 3.
- (E) The acceleration of point D on the rim of the inner pulley at  $t = 0$  is greater than  $1000 \text{ mm/s}^2$ .
- (F) None of the previous statements is correct.

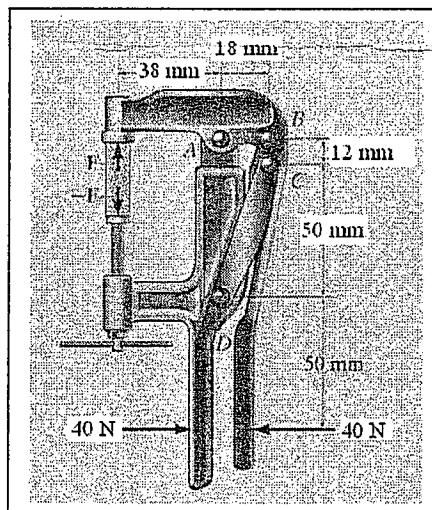


科目：應用力學【機電系碩士班乙組、丙組】

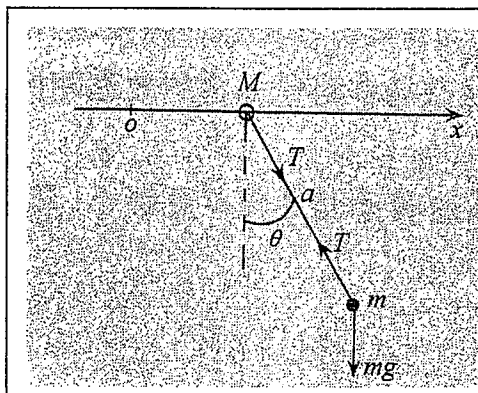
4. The disk has a radius of  $r = 0.3\text{-m}$  and a mass of  $8\text{-kg}$ . If the disk is released from rest on a  $\theta = 30^\circ$  inclined plane, determine the angular acceleration of the disk. The coefficients of static and kinetic friction between the disk and inclined plane are  $\mu_s = 0.15$  and  $\mu_k = 0.1$ , respectively. (25%)



5. If a force of  $40\text{ N}$  is applied to the grip of the clamp, determine the compressive force  $F$  that the wood block exerts on the clamp and the force at each pin. (25%)

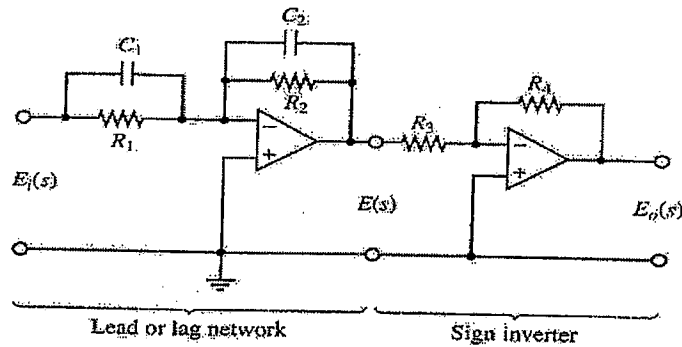


6. 一個質量為  $M$  的小球套在光滑的水平的金屬絲上。另一個質量為  $m$  的質點通過長度  $a$  無質量的不可伸縮的繩子連在小球上。使繩子沿金屬絲拉直然後釋放，求此後繩子張力  $T$  與繩子、垂直線夾角  $\theta$  的關係。(25%)



科目：自動控制【機電系碩士班丙組】

1. (10%) If the transfer function  $E_o(s)/E_i(s)$  shown in Fig. 1 is  $K \frac{s + (1/b)}{s + (1/ab)}$ ,
  - (a) (5%) determine the parameters  $K, a, b$  in terms of  $R_1, R_2, R_3, R_4, C_1, C_2$ .
  - (b) (5%) How to determine this network is a lead or lag network?



2. (20%) The block diagram of a linear system is shown in Fig. 2.
  - (a) (10%) Construct a parameter plane ( $K_1$  versus  $K_2$ ) to show the stable region.
  - (b) (5%) Find the steady state error in terms of  $K_1$  and  $K_2$  when a ramp function input  $r(t) = tu_s(t)$  is applied.
  - (c) (5%) Find the values of  $K_1$  and  $K_2$  so that the relative damping ratio  $\zeta$  of the complex roots of the characteristic equation is 0.5 and the rise time  $t_r$  of the unit-step response is approximately 1 sec. (Hint: using  $t_r = (1 - 0.4167\zeta + 2.917\zeta^2)/\omega_n$ )

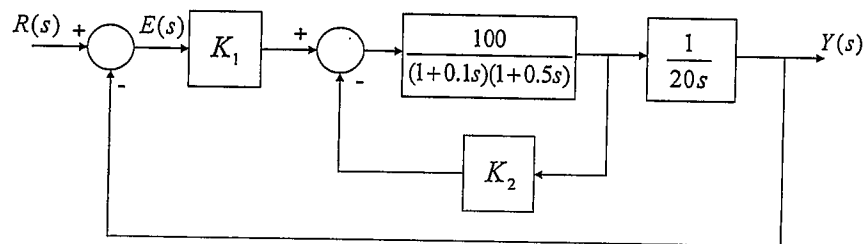


Fig. 2

3. (20%) Consider the open-loop transfer function for the system is

$$G(s) = \frac{K(s+2)}{s(s-1)(s^2+6s+36)}$$

- (a) (5%) Find the range of  $K$  for stability.
- (b) (5%) Determine the asymptotes of the root loci.
- (c) (5%) Derive that points  $s=0.47$  and  $s=-3.9$  on the root loci are breakaway and break-in points, respectively.
- (d) (5%) Sketch the complete root loci of the system.

科目：自動控制【機電系碩士班丙組】

4. (10%) Please discuss the meanings of the following terminologies, from the viewpoint of controller design: (a) phase margin and gain margin, (b) minimum phase system and non-minimum phase system, (c) system with transport lag, (d) M-circle and N-circle, (e) bandwidth.
5. (10%) The Bode diagrams of two dynamic systems are shown in Fig. 3 and Fig. 4. Please guess the possible transfer functions of the systems. Discuss your answers in detail.

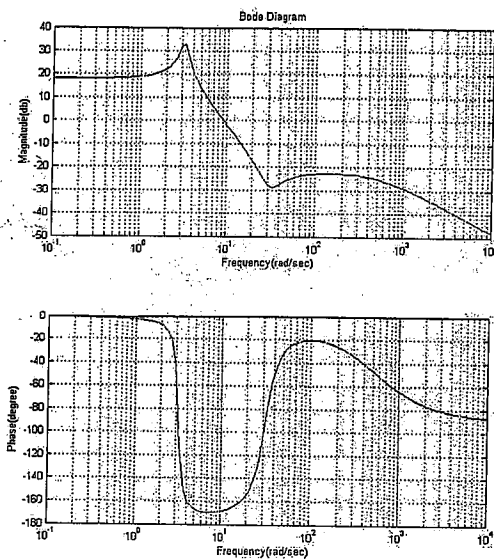


Fig. 3

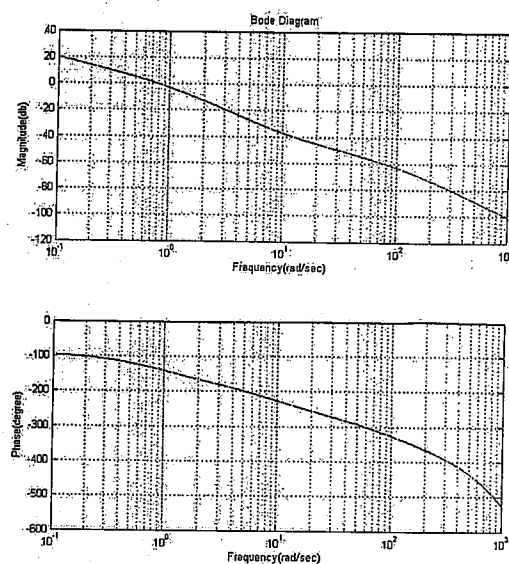


Fig. 4

6. (20%) Consider a unit feedback control system as shown in Fig.5. Please draw a desired polar plot of the open loop transfer function  $G(s)C(s)$  that the closed-loop system has a satisfactory stability and has a bandwidth more than 100 rad/sec. Discuss your plot in detail.

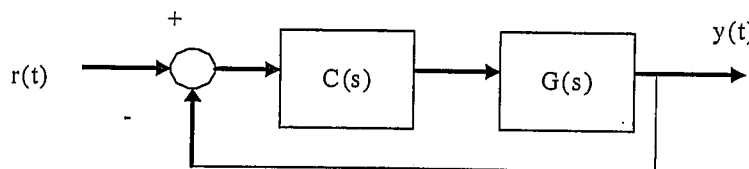


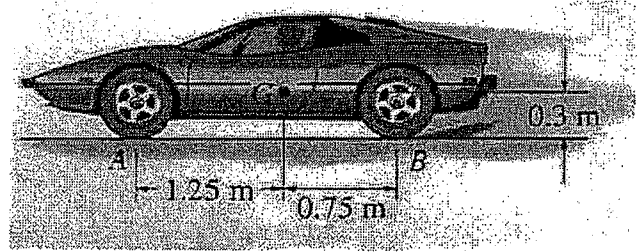
Fig. 5

7. (10%) Please discuss your procedure or approach of applying the lead compensator in improving the stability of a control system.

科目：動力學【機電系碩士班丁組、戊組】

Problems 1-13: (單選題，每題 5 分) 請在答案卷作答

The car shown in the right-hand side has a mass of 2 Mg and a center of mass at G. Assume that the "driving" wheels in the back are always slipping, whereas the front wheels freely rotate. Neglect the mass of the wheels. The coefficient of kinetic friction between the wheels and the road is 0.25 ( $\mu_k = 0.25$ ). Answer the following questions.



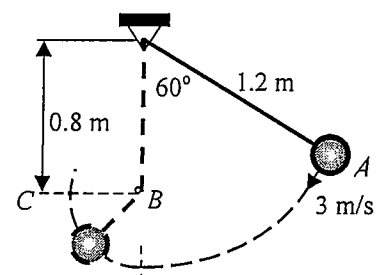
- In drawing the free-body diagram, how many forces do you expect acting at the front wheel at point A?  
(a) 1, (b) 2, (c) 3, (d) 4, or (e) 5.
- How many forces do you expect acting at car? (a) 1, (b) 2, (c) 3, (d) 4, or (e) 5.
- What is the car's acceleration? (a)  $0.79 \text{ m/s}^2$ , (b)  $1.19 \text{ m/s}^2$ , (c)  $1.27 \text{ m/s}^2$ , (d)  $1.59 \text{ m/s}^2$ , or (e)  $1.99 \text{ m/s}^2$ .
- What is the normal force acting on the front wheel?  
(a) 1.25 kN, (b) 6.88 kN, (c) 12.7 kN, (d) 18.4 kN, or (e) 19.6 kN.

A racing car travels around the horizontal circular track that has a radius of 300 ft. The car increases its speed at a constant rate of  $7 \text{ ft/s}^2$ , starting from rest. Answer the following questions.

- What is the time needed for it to reach an acceleration of  $8 \text{ ft/s}^2$ ?  
(a) 2.87 s, (b) 3.87 s, (c) 4.87 s, (d) 5.87 s, or (e) 6.87 s.
- What is the speed at this instant? (a) 13.1 ft/s, (b) 19.1 ft/s, (c) 20.1 ft/s, (d) 27.1 ft/s, (e) 34.1 ft/s.
- What is the normal acceleration at this instant?  
(a)  $0.163 \text{ ft/s}^2$ , (b)  $1.35 \text{ ft/s}^2$ , (c)  $2.45 \text{ ft/s}^2$ , (d)  $3.87 \text{ ft/s}^2$ , or (e)  $10.63 \text{ ft/s}^2$ .
- A harmonic motion has an amplitude of 0.05 m and a frequency of 10 Hz. What is the period?  
(a) 0.05 s, (b) 0.1 s, (c) 0.2 s, (d) 0.5 s, (e) 1 s.
- In Problem 8, what is the maximum acceleration?  
(a)  $39.48 \text{ m/s}^2$ , (b)  $78.96 \text{ m/s}^2$ , (c)  $157.91 \text{ m/s}^2$ , (d)  $197.39 \text{ m/s}^2$ , (e)  $394.79 \text{ m/s}^2$ .
- The maximum amplitude and the maximum acceleration of the foundation of a centrifugal pump were found to be  $x_{\max} = 0.25 \text{ mm}$  and  $\ddot{x}_{\max} = 0.4 g$  for its harmonic oscillation. What is the operating speed of the pump?  
(a) 5 rpm, (b) 10 rpm, (c) 15 rpm, (d) 20 rpm, (e) 40 rpm.
- An industrial press is mounted on a rubber pad to isolate it from its foundation. If the rubber pad is compressed 5 mm by the self-weight of the press, what is the natural frequency of the system?  
(a) 3 Hz, (b) 4 Hz, (c) 5 Hz, (d) 6 Hz, (e) 7 Hz.
- The natural frequency of a spring-mass system is found to be 2 Hz. When an additional mass of 1 kg is added to the original mass  $m$ , the natural frequency is reduced to 1 Hz. What is the original mass?  
(a) 0.33 kg, (b) 0.66 kg, (c) 1.32 kg, (d) 1.65 kg, (e) 1.98 kg.
- In Problem 12, what is the spring constant  $k$ ?  
(a) 12.57 N/m, (b) 22.68 N/m, (c) 52.64 N/m, (d) 75.42 N/m, (e) 100.56 N/m.

Problems 14-18: (單選題，每題 7 分) 請在答案卷作答

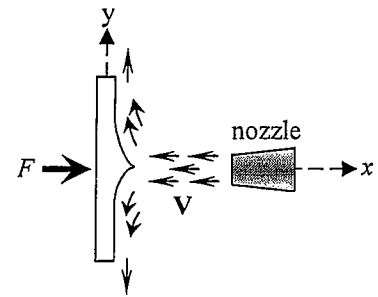
- The ball is released from position A with a velocity of 3 m/s and swings in a vertical plane. At the bottom position, the cord strikes the fixed bar at B, and the ball continues to swing in the dotted arc. Calculate the velocity  $v_c$  of the ball as it passes position C.  
(a) 2.56 m/s, (b) 3.59 m/s, (c) 4.58 m/s, (d) 5.12 m/s, (e) 5.33 m/s.



(Continued)

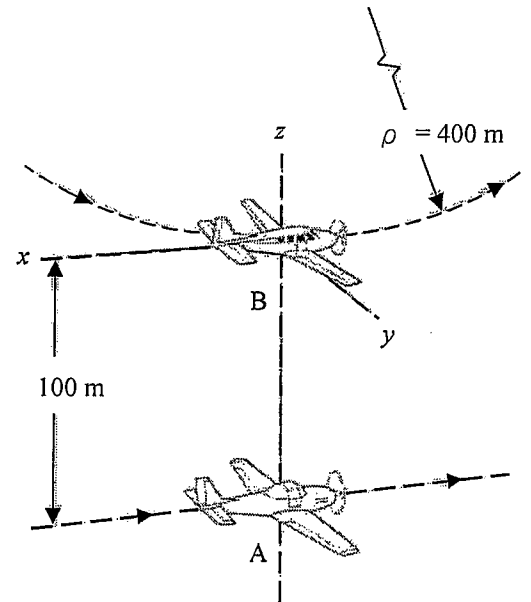
科目：動力學【機電系碩士班丁組、戊組】

15. A jet of air issues from the nozzle with a velocity ( $V$ ) of 100 m/s at the rate of  $0.2 \text{ m}^3/\text{s}$  and is deflected by the right-angled vane. Calculate the force  $F$  required to hold the vane in a fixed position. The density of the air is  $1.206 \text{ kg/m}^3$ .



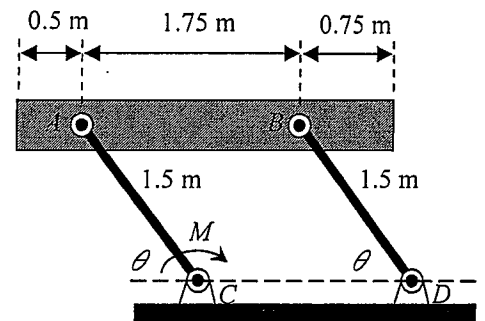
- (a) 25.5 N, (b) 24.1 N, (c) 23.12 N, (d) 21.56 N, (e) 19.33 N.

16. Aircraft B has a constant speed of 540 km/hr at the bottom of a circular loop of 400-m radius ( $\rho$ ). Aircraft A flying horizontally in the plane of the loop passes 100 m directly under B at a constant speed of 360 km/hr. With coordinate axes attached to B as shown, determine the acceleration that A appears to have to the pilot of B for this instant.



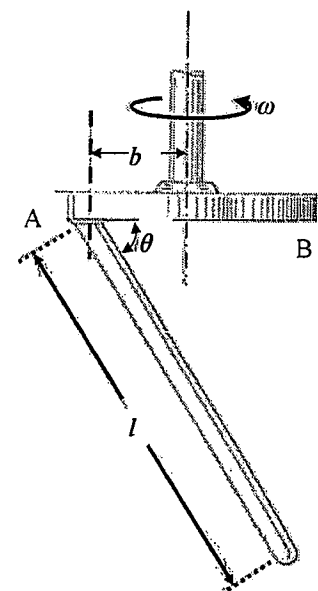
- (a)  $-4.69\mathbf{k} \text{ m/s}^2$ , (b)  $-5.76\mathbf{k} \text{ m/s}^2$ , (c)  $-6.89\mathbf{k} \text{ m/s}^2$ , (d)  $-7.62\mathbf{k} \text{ m/s}^2$   
(e)  $-8.34\mathbf{k} \text{ m/s}^2$ .

17. The uniform 200-kg bar  $AB$  is raised in the vertical plane by the application of a constant couple  $M = 3 \text{ kN} \cdot \text{m}$  applied to the hinged link at  $C$ . The mass of the links is small and may be neglected. If the bar starts from rest at  $\theta = 0$ , determine the magnitude of the force supported by the pin at  $A$  as the position  $\theta = 60^\circ$  is passed.



- (a) 1.34 kN, (b) 2.02 kN, (c) 3.42 kN, (d) 4.75 kN, (e) 5.63 kN.

18. The uniform slender rod of length  $l$  is welded to the bracket at  $A$  on the underside of the disk  $B$ . The disk rotates about a vertical axis with a constant angular velocity  $\omega$ . Determine the value of  $\omega$  that will result in a zero moment supported by the weld at  $A$  for the position  $\theta = 60^\circ$  with  $b = l/4$ .



- (a)  $2\sqrt{\frac{\sqrt{3}g}{l}}$ , (b)  $4\sqrt{\frac{\sqrt{5}g}{l}}$ , (c)  $\sqrt{\frac{2g}{3l}}$ , (d)  $\sqrt{\frac{\sqrt{5}g}{l}}$ , (e)  $\sqrt{\frac{3g}{2l}}$ .



科目：靜力學【機電系碩士班丁組、戊組】✓

請在答案卷上作答

- 有一腳踏式打氣筒欲將輪胎打氣至 70 lb，其相關尺寸與角度如圖 1 (Figure 1) 所示。(A、B、C 點均為 pin connection)

  - 請畫出該系統之自由體圖(Free-Body Diagram)。(3%)
  - 請寫出平衡方程式(Equations of Equilibrium)。(3%)
  - 請求出垂直力 P 為多少。(4%)
- The 100-mm diameter pulley as shown in Fig. 2 fits loosely on a 10-mm-diameter shaft. ( $\mu_s = 0.4$ ) Determine the minimum tension T in the belt need to

  - Raise the 100-kg block. (5%)
  - Lower the block? (5%)

Assume that no slipping occurs between the belt and pulley and neglect the weight of the pulley.

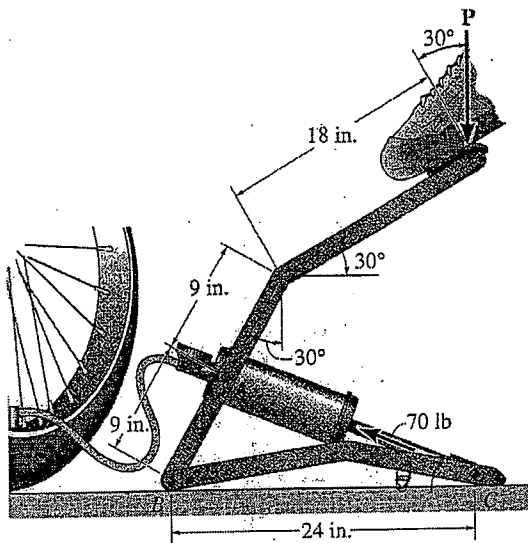


Figure 1

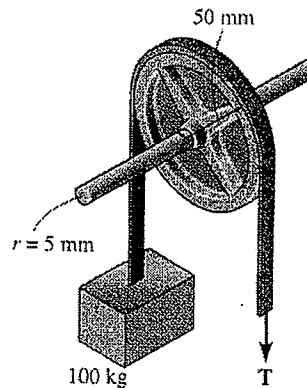


Figure 2

- 試求 Figure 3 所示之工字鐵的形心(centroid)之  $\bar{y}$  值(5%)
  - 請問質心(center of mass)和重心(center of gravity)的定義，以及在何種狀況下，同一剛體的質心及重心兩點不重合？(5%)

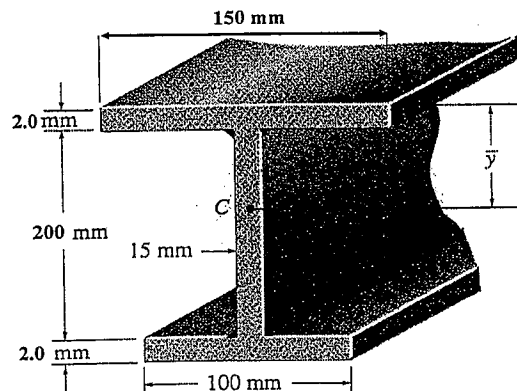


Figure 3

科目：靜力學【機電系碩士班丁組、戊組】

4. Figure 4 shows one of the Great Pyramids. Dimensions are shown
- Write down the position vectors of points A and B relative to O, expressing your answer as components in the basis shown. (5%)
  - Find the area of the triangle OAB (5%)
  - Find a unit vector perpendicular to the face OAB. (5%)
  - The face OAB is subjected to a force per unit area of  $100Nm^{-2}$  due to wind load. The force acts in a direction perpendicular to OAB. Find the total force acting on the face, expressing your answer as components in the basis shown. (5%)

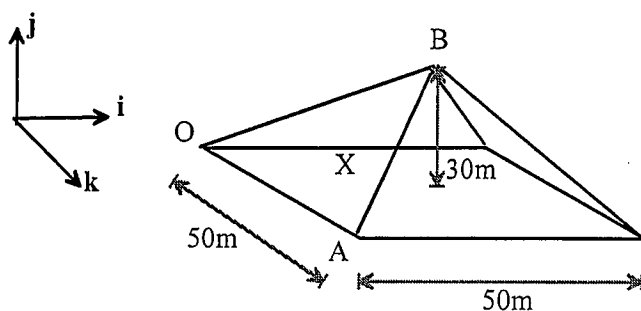


Figure 4

5. An eye bolt is screwed in perpendicular to a wall in the  $(j, k)$  plane so that 1 cm of its length extends outside the wall at point A. A cable attached to the end of the eye bolt at O runs over a frictionless pulley and supports the weight W, as shown in Figure 5. The bolt will pull out of the wall if the component of the force perpendicular to the wall exceeds 100N.
- What value is the maximum value to the weight W that the structure can support without pulling the bolt from the wall? (5%)
  - If the weight of W is 45N, what value is the moment about point A due to loading on the bolt? (5%)

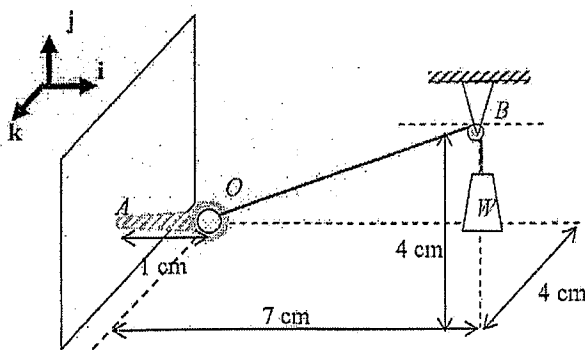


Figure 5

科目：靜力學【機電系碩士班丁組、戊組】

6. The car has weight  $W$  which acts at its center of mass,  $G$ . Moving at constant speed, an aerodynamic downward force  $F=0.1W$  acts at the spoiler as shown in Figure 6. What value is the reaction force at the rear wheels? (10%)

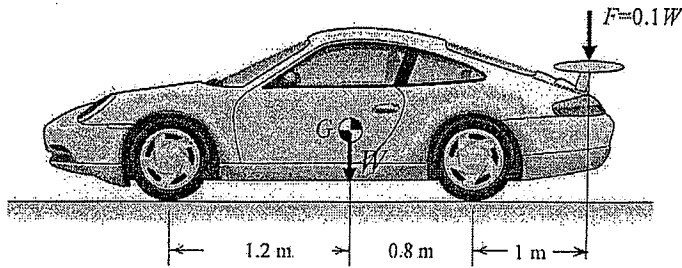


Figure 6

7. Each bar shown in Figure 7 is of weight  $W$ , and the spring is unstretched when  $\alpha = 90^\circ$ .
- (a) Show that the system is in equilibrium when  $\alpha = \sin^{-1}(W/4kL)$ . (20%)
- (b) Is the equilibrium position described in (a) stable? (10%)

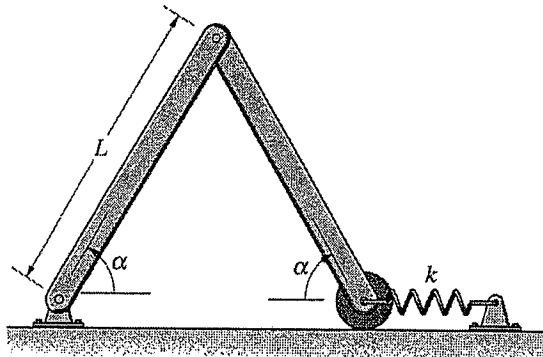


Figure 7