工程數學 機電碩

一.

(1) (15%) Solve the first order ODE of $y' = 4y - 2y^2$ and explain its physical meaning by sketching the general solution.

(2) (10%) Find the length of the following curve: $\mathbf{r}(t) = [2\cos t, 2\sin t, 6t]$ from (2, 0, 0) to (2, 0, 24π).

二.

(1) (9%) Find the eigenvalues and eigenfunctions of the following problem

$$y'' + \lambda y = 0, \quad y(0) = 0, \quad y(1) + y'(1) = 0.$$

(2) (16%) Evaluate (showing details)

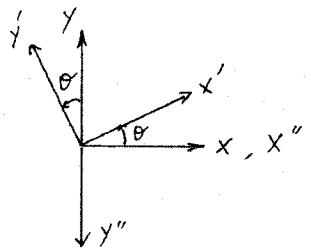
$$\text{(a) } \int_0^{2\pi} \frac{\sin\theta}{3+\cos\theta} d\theta; \text{ (b) } \int_{-\infty}^{\infty} \frac{1}{x^4+16} dx.$$

三.

(1) (9%) Give the definition of (a) rank of a matrix, (b) basis of a vector space, (c) unitary matrix.

(2) (12%) Try to transform a quadratic form $U^T AU = 1, U = \begin{Bmatrix} x \\ y \end{Bmatrix}, A = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$, into standard equation of an ellipse $\left(\left(\frac{P_1}{R_1}\right)^2 + \left(\frac{P_2}{R_2}\right)^2 = 1\right)$.

(3) (4%) Give the transformation matrices [A] and [B] of $\begin{Bmatrix} x' \\ y' \end{Bmatrix} = [A] \begin{Bmatrix} x \\ y \end{Bmatrix}$ and $\begin{Bmatrix} x'' \\ y'' \end{Bmatrix} = [B] \begin{Bmatrix} x \\ y \end{Bmatrix}$ of the coordinate systems below.



四. 簡答題部分 (每題 5 分，共計 25 分)

The following five problems do not require any rigorous mathematical proof. All you have to do is to explain the reason behind your answers clearly.

(1) (5%) With matrix $\boldsymbol{A}$ and vector $\boldsymbol{b}$ given, one can find $\boldsymbol{x}$ to satisfy the system of linear algebraic equations $\boldsymbol{Ax} = \boldsymbol{B}$ only when $\boldsymbol{A}$ is a nonsingular square matrix. Is this statement correct? Why or why not?

(2) (5%) By taking the Laplace transform of differential equation $y'(t) + 3y(t) = f(t)$, Mr. Chen has determined the solution for this differential equation to be the inverse Laplace transform of $Y(s) = F(s)/(s+3)$. Is this solution satisfactory?

(3) (5%) If $f(t)$ is not a continuous function, then can you perform Fourier series expansion for this function? Why or why not?

(4) (5%) Can we use Laplace transform to solve nonlinear differential equations? Why or why not?

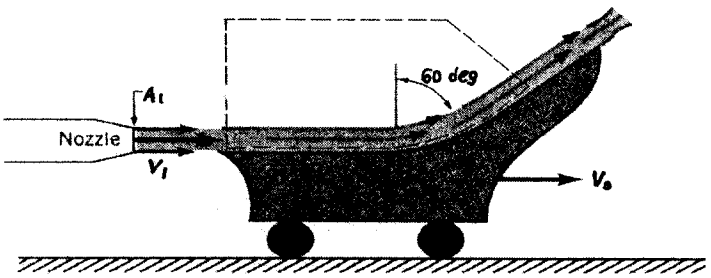
(5) (5%) For functions $f(t) = f(t+T)$ and $g(t) = g(t+T)$, it has been found that the waveform of $f(t)$ is smoother than that of $g(t)$. As a result, we expect the Fourier series expansion of $f(t)$ converges faster than that of $g(t)$. Can you explain why?

1. (10 %)
(a) Give the definition of a fluid?
(b) List the basic dimensions used to describe the problem of a hot jet flow and the corresponding units in SI system.
(c) What is newtonian fluid?
(d) Does each of the following fluids belong to newtoniam fluid? water, air.
(e) Under what condition the pressure at a point is independent of direction?

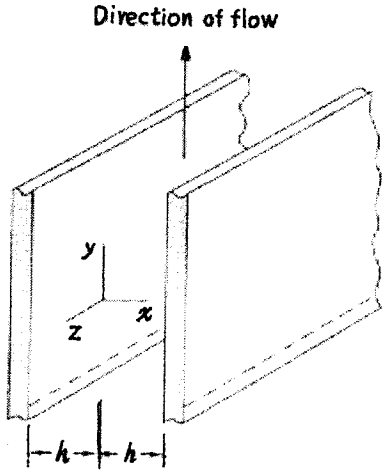
2. (10 %)
The velocity field of a flow is given by u=-y, and v=x.
(a) Is the flow steady or unsteady?
(b) Determine the equation of the streamlines.
(c) Draw the streamline passing though the point (x, y)=(1, 0) in the x-y plane.
(d) Is the flow rotational?
(e) Determine the acceleration of a fluid element passing through the point (x,y)=(3,2)

3. (5 %, both (a) and (b) have to be answered correctly to obtain the 5 points)
There are two general approaches in analyzing fluid mechanics problems, the Eulerian method, and the Lagrangian method. Which method does each of the following descriptions correspond to?
(a) Some ornithologists (鳥類學家) study bird migrations by obtaining the data of the rate at which birds pass a certain location on their migration route.
(b) Some ornithologists study bird migrations by tagging certain birds with radio transmitters and following their motion along the migration route.

4. (10 p%)
A vane on wheels moves with constant velocity V_o when a stream of water having a nozzle exit velocity V_1 is turned 30° by the vane (i.e. the exit velocity has an angle of 60° with respect to the vertical line) as indicated in the figure. Determine the magnitude and direction of the force, F, exerted by the stream of water on the vane surface. The speed of the water jet leaving the nozzle is 30 m/s, and the vane is moving to the right with a constant speed of 6m/s. We assume that the water flow is frictionless and the change in water elevation across the vane is negligible. The speed of the water relative to the vane is constant. The water density is 1000kg/m^3. The water flow on the vane maintains a constant cross-sectional area A_1=0.0006 m^2.



5. (15 %)
A viscous, incompressible fluid flows between the two infinite, vertical, parallel plate of the following figure. Pressure gradient exists in the direction of the flow, $\partial p/\partial y$. Assume that the flow is laminar, steady, and uniform. The velocity in the y-direction, v, is a function of x. The gravitational force is in the negative y-direction, $g_y=-g$. The viscosity of the fluid is μ, and the density is ρ. The distance between the two plates is 2h.
(a) Write down the Navior-Stokes equation in the y-direction. i.e. the differential equation for the velocity v.
(b) What are the boundary conditions for (a)?
(c) Solve for the velocity v.
(d) Find the equation to relate the pressure gradient and the flow rate per unit width in the z-direction.



6. Please "define" and "explain" the followings :
(a) Grashof number , (b) Biot number, (c) thermal boundary layer, (d) Boussineq approximation. (12 %)
7. Please "draw" and "explain" the velocity profiles of a laminar flow in circular tube for (a) hydrodynamic entrance region, (b) fully developed region. (6 %)
8. For a steady, incompressible laminar flow over a flat plate, (a) write down the "continuity", "momentum", and "energy" equations,(9 %) (b) by assuming constant fluid properties, negligible viscous dissipation, and no pressure gradient, write down the "momentum" and "energy" equations,(6 %) (c) explain how to achieve the "Blasius solution" (5 %)
9. For flow over a flat plate of length "L", the local heat transfer coefficient is known to vary as $x^{(-1/2)}$, where x is the distance from the leading edge of the plate. What is the ratio of the "average" Nusselt number for the entire plate to the "local" Nusselt number at x = L . (12 %)

Thermodynamics: (65%)

1. (6%). When an isolated system undergoes a reactive process, the system properties such as internal energy (U), entropy (S), and exergy (X) will change or remain constant. Please select the correct change condition of this isolated reactive system?
   (a) $\Delta U = 0, \Delta S \leq 0, \Delta X = 0$, (b) $\Delta U \geq 0, \Delta S \geq 0, \Delta X \leq 0$, (c) $\Delta U = 0, \Delta S \leq 0, \Delta X \leq 0$,
   (d) $\Delta U = 0, \Delta S \geq 0, \Delta X \leq 0$, (e) $\Delta U \geq 0, \Delta S \geq 0, \Delta X = 0$.

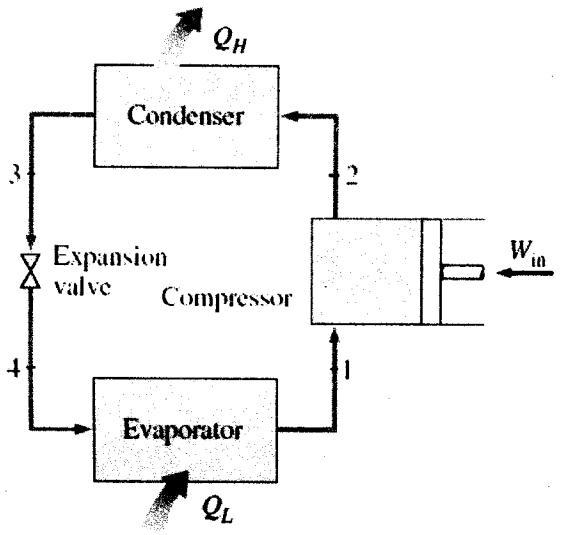
2. (8%). An open system is interactive with its surroundings through the three transfer mechanisms mass, heat and work. For the three mechanisms what can cause the changes of the internal energy, the entropy, and the exergy of a control volume. Please discuss each type of the change, respectively?

3. (6%). What is enthalpy of formation? How does it differ from the enthalpy of combustion? When are the enthalpy of formation and the enthalpy of combustion identical?

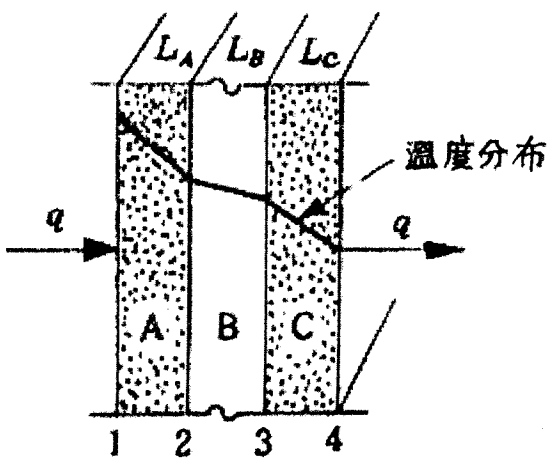
4. (15%). An air-standard cycle is executed in a closed system with 0.005 kg of air and consists of the following three processes:
   1-2 Isentropic compression from 100 kPa and 27°C to 1 MPa
   2-3 P = *constant* heat addition in the amount of 3.45 kJ
   3-1 $P = c_1 v + c_2$ heat rejection to initial state (c_1 and c_2 are constants)
   (*a*) Show the cycle on *P-v* and *T-s* diagrams.
   (*b*) Calculate the heat rejected.
   (*c*) Determine the thermal efficiency.
   Assume constant specific heats at room temperature.
   (Note: air at room temperature are $c_p = 1.005$ kJ/kg.K, $c_v = 0.718$ kJ/kg·K, and $k = 1.4$)

5. (15%). Derive relations for the internal energy change and the entropy change as a gas that obeys the equation of state $(P+a/v^2)v = RT$. Assume that in the range of interest c_v varies according to the relation $c_v = c_1 + c_2 T$, where c_1 and c_2 are constants.
   (Note: $du = c_v dT + [T(\partial P/\partial T)_v - P]\, dv$, and $ds = (c_v/T)dT + (\partial P/\partial T)_v\, dv$)

6. (15%) 理想蒸氣壓縮冷凍循環如下圖(1)所示，根據該圖回答下列問題.
   (a) Draw a T-s diagram. Describe each process for the cycle.
   (b) How do you calculate heat and work for each process of the cycle?
   (c) Which process provides refrigeration effect?
   (d) What is the COP of the refrigeration cycle?
   (e) If the cycle is used as a heat pump, what is the COP?



圖(1) 理想蒸氣壓縮冷凍循環



圖(2) 一維熱傳導

Heat Conduction and Radiation: (35%)

7. (a) (10%) 如上圖(2)所示，材料 A、B、C 的熱阻(R_A、R_B、R_C)各爲何? 總熱阻(R_t)爲何? 那一個材料中的熱傳量最大?

   (b) (10%) 寫出熱傳 q 與下列各組溫度的關係 (1)T_1、T_2 (2)T_2、T_3 (3)T_3、T_4 (4)T_1、T_3 (5) T_1、T_4

   (c) (8%) 如何運用上圖一維熱傳導的原理於實驗量測物質 B 的熱傳導係數?

   (2%) 熱傳導係數的公制單位爲何?

8. (5%) What is the basic law for thermal radiation?

Prob.#1 (25%)

Please choose the correct answers for problem (1) to problem (3).

Please be noted that the correct answers for each problem may be more than one.

(1) Which following statements are correct? (5%)

(A) When a 4-wheels driving automobile moves to right, the directions of friction forces at the wheels are directed to right also.

(B) When a disk rolls on a rough surface and sliding is impending, the magnitude of the friction force F equals to $\mu_k N$, where μ_k is the coefficient of kinetic friction & N is the magnitude of the normal force.

(C) The contacting particles of two rigid bodies that are in contact have equal components of velocityin the direction of the line tangent to the surface of the bodies at their point of contact.

(D) When a body executes plane motion rolls without sliding on a fixed surface, the point of contact is the instantaneous center for the rolling body, so both velocity and acceleration of the particle of the body at the contact point are zero.

(E) The total mechanical energy of a freely vibrating damped system is constant.

(F) None of the previous statements is correct.

(2) A uniform sphere of mass m and radius r is projected along a rough horizontal surface with a linear velocity V_0 and no initial angular velocity. Denoting by μ_k the coefficient of kinetic friction between the sphere and the floor, g is the acceleration of gravity, and the mass moment of inertia of the sphere about centroidal axis is $(2mr^2/5)$. Which following statements are correct? (10%)

(A) When the sphere rolls with sliding, the magnitude of the linear acceleration of the center of the sphere depends on m.

(B) When the sphere rolls with sliding, the magnitude of the angular acceleration of the sphere does not depend on m.

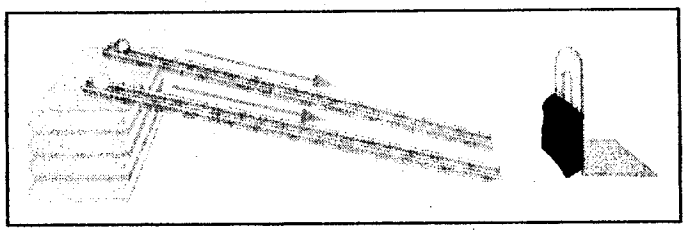
(C) The time t_1 at which the sphere will start rolling without sliding equals to $(2/7)(V_0/\mu_k g)$.

(D) The linear velocity of the center of the sphere at time t_1 equals to $(2/7)(V_0)$.

(E) The angular velocity of the sphere at time t_1 equals to $(2/7)(V_0/r)$.

(F) None of the previous statements is correct.

(3) Two spheres ($r_A = r_B = r$, $m_A = m$, $m_B = 3m$) rolling down along a frictionless incline at the same time and striking into a clay plate making two holes in the plate. Which following statements are correct? (10%)



(A) If both the balls slide without rolling, sphere B will strike into the plate first.

(B) If both the balls slide without rolling, the depths of both holes are the same.

(C) If both the balls slide without rolling, the velocity of each ball depends on its mass.

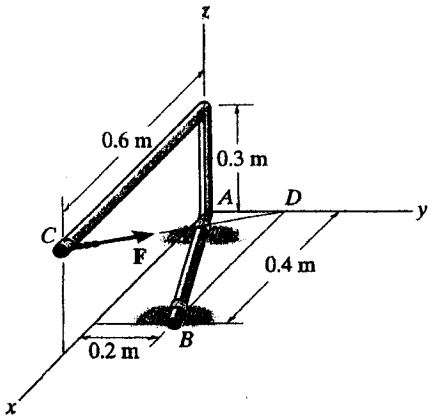
(D) If both the balls roll without sliding, both spheres will strike into the plate at same time.

(E) If both the balls roll without sliding, the depth of the hole stricken by sphere B is deeper.

(F) None of the previous statements is correct.

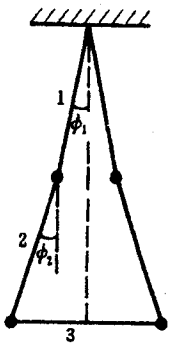
Prob.#2 (25%)

The rod shown in the figure is supported by two brackets at *A* and *B*. (a) Determine the moment $\vec{M}_{AB}$ produced by $\vec{F}=\{-600\hat{i}+200\hat{j}+300\hat{k}\}$ N, which tends to rotate the rod about the *AB* axis. (b) Determine the angle between force $\vec{F}$ and line *CB*.



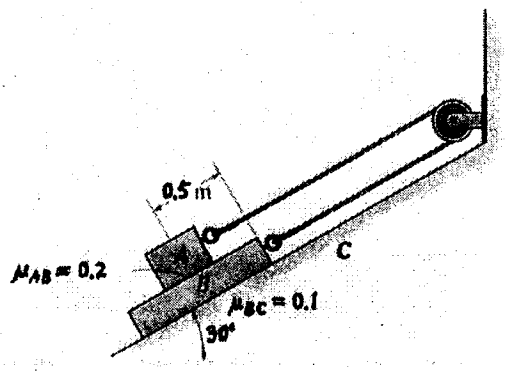
Prob.#3 (25%)

Five thin uniform rods, with mass m and length ℓ, are connected by frictionless hinges and loaded by their own weight as shown in the following figure. As the rods are in balance as shown in the figure, ϕ_1 and ϕ_2 are the angle between the rod and the direction of gravity. Please give the algebraic equations which $\sin(\phi_1)$ and $\sin(\phi_2)$ satisfy.



Prob.#4 (25%)

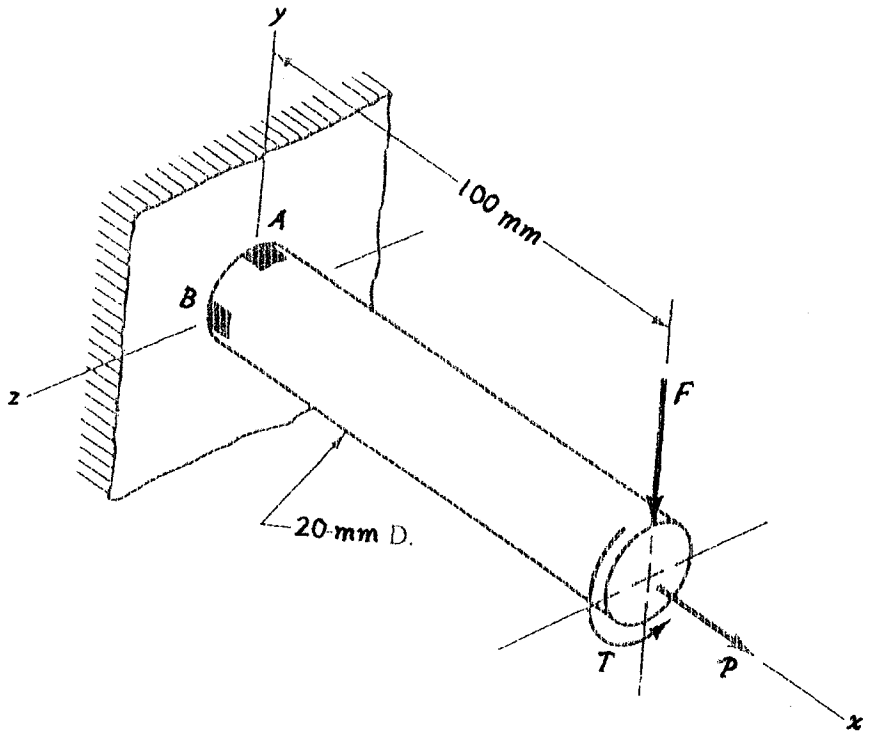
The 10-kg block A rests on the 50-kg plate B in the position shown. Neglecting the mass of the rope and the pulley, and using the coefficients of kinetic friction indicated, determine the time needed for block A to slide 0.5 m on the plate when the system is released from rest.



Part I: To select the proper answer for following questions (5% for each)

乙
4-1

(1) Consider a rod is loaded by forces $F = 0.55\ kN$, $P = 8.0\ kN$, and $T = 30\ Nm$.



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1. ( ) *The stress tensor at the point **A** , as shown in above figure, is*

(A) $[\sigma_{ij}] = \begin{bmatrix} 25.5 & 0 & -21.4 \\ 0 & 0 & 0 \\ -21.4 & 0 & 0 \end{bmatrix}$ (MPa) (B) $[\sigma_{ij}] = \begin{bmatrix} 95.5 & 0 & 19.1 \\ 0 & 0 & 0 \\ 19.1 & 0 & 0 \end{bmatrix}$ (MPa)

(C) $[\sigma_{ij}] = \begin{bmatrix} 25.5 & -21.4 & 0 \\ -21.4 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (MPa) (D) $[\sigma_{ij}] = \begin{bmatrix} 95.5 & 19.1 & 0 \\ 19.1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (MPa)

(E) *None*

2. ( ) *The stress tensor at the point **B** , as shown in above figure, is*

(A) $[\sigma_{ij}] = \begin{bmatrix} 25.5 & 0 & -21.4 \\ 0 & 0 & 0 \\ -21.4 & 0 & 0 \end{bmatrix}$ (MPa) (B) $[\sigma_{ij}] = \begin{bmatrix} 95.5 & 0 & 19.1 \\ 0 & 0 & 0 \\ 19.1 & 0 & 0 \end{bmatrix}$ (MPa)

(C) $[\sigma_{ij}] = \begin{bmatrix} 25.5 & -21.4 & 0 \\ -21.4 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (MPa) (D) $[\sigma_{ij}] = \begin{bmatrix} 95.5 & 19.1 & 0 \\ 19.1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (MPa)

(E) *None*

3. ( ) *The principal stresses at the point A is*

(A) $\sigma_1 = 99.2;\ \sigma_2 = 0$ *and* $\sigma_3 = -3.7\ MPa$ (B) $\sigma_1 = 99.2;\ \sigma_2 = 13.7$ *and* $\sigma_3 = 9.2\ MPa$

(C) $\sigma_1 = 37.7; \sigma_2 = 0$ *and* $\sigma_3 = -12.2\ MPa$ (D) $\sigma_1 = 37.7; \sigma_2 = 12.2$ *and* $\sigma_3 = 0.\ MPa$

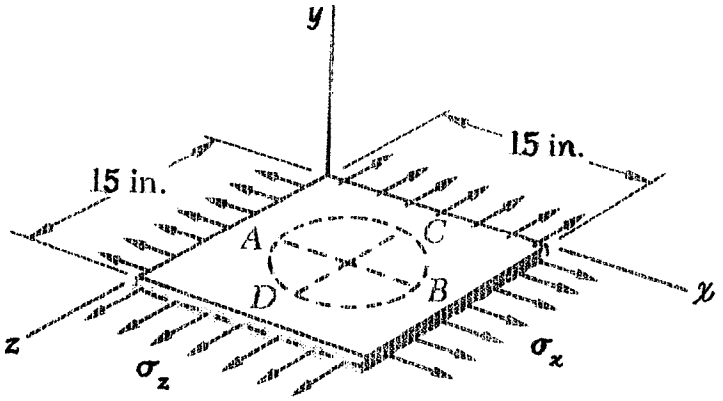
(E) *None*

4. ( ) *The maximum shear stress at the point **A** is*

(A) $\tau_{max} = 24.95\ MPa$ (B) $\tau_{max} = 51.45\ MPa$ (C) $\tau_{max} = 45.0\ MPa$

(D) $\tau_{max} = 18.85\ MPa$ (E) *None*

(2) A circle of diameter $d = 9\ mm.$ is scribed on an unstressed aluminum plate of thickness $t = 0.75\ in.$ Forces acting in the plane of the plate later cause normal stresses $\sigma_x = 12\ ksi$ and $\sigma_z = 20\ ksi$. Knowing that the plate is made of aluminum which $E = 10 \times 10^3\ ksi$ and Poison's ratio $\nu = 1/3$. It is also noted $\sigma_y = 0$.



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5. ( ) *The corresponding strains in the coordinate directions are*

(A) $\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\end{Bmatrix} = \begin{Bmatrix}+0.533\\ +1.600\\ -1.067\end{Bmatrix} \times 10^{-3}\ in/in$ (B) $\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\end{Bmatrix} = \begin{Bmatrix}-0.533\\ -1.067\\ +1.600\end{Bmatrix} \times 10^{-3}\ in/in$

(C) $\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\end{Bmatrix} = \begin{Bmatrix}+0.533\\ -1.067\\ -1.600\end{Bmatrix} \times 10^{-3}\ in/in$ (D) $\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\end{Bmatrix} = \begin{Bmatrix}+0.533\\ -1.067\\ +1.600\end{Bmatrix} \times 10^{-3}\ in/in$ (E) *None*

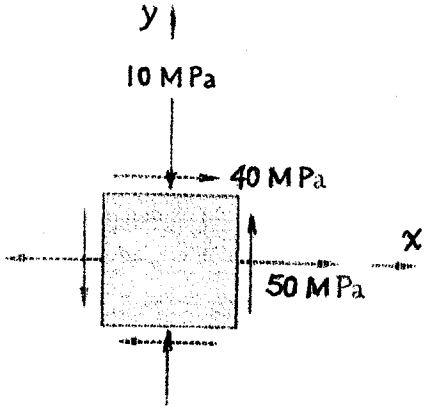
6. ( ) *The change in the length of diameter AB is*

(A) $+4.8 \times 10^{-3} in.$ (B) $+8.8 \times 10^{-3} in.$ (C) $+14.4 \times 10^{-3} in.$
(D) $+20.6 \times 10^{-3} in.$ (E) *None*

7. ( ) *The change in the thickness of the plate is*

(A) $-0.048 \times 10^{-3} in.$ (B) $-0.800 \times 10^{-3} in.$ (C) $-1.440 \times 10^{-3} in.$
(D) $-10.366 \times 10^{-3} in.$ (E) *None*

(3) *For the state of plane stress shown in following figure*



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8. ( ) *The corresponding stress tensor is*

(A) $\begin{bmatrix} 50 & -40 & 0 \\ -40 & 10 & 0 \\ 0 & 0 & 0 \end{bmatrix}(MPa)$ (B) $\begin{bmatrix} 50 & 40 & 0 \\ 40 & -10 & 0 \\ 0 & 0 & 0 \end{bmatrix}(MPa)$

(C) $\begin{bmatrix} 50 & -40 & 0 \\ -40 & -10 & 0 \\ 0 & 0 & 0 \end{bmatrix}(MPa)$ (D) $\begin{bmatrix} -50 & 40 & 0 \\ 40 & -10 & 0 \\ 0 & 0 & 0 \end{bmatrix}(MPa)$ (E) *None*

9. ( ) *The maximum principal stress is*

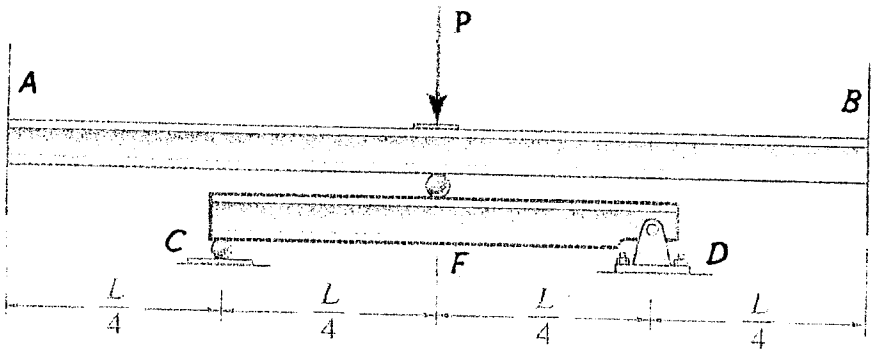
(A) $\sigma_{max} = 70.0\ MPa$ (B) $\sigma_{max} = 50.0\ MPa$ (C) $\sigma_{max} = 40.0\ MPa$
(D) $\sigma_{max} = 30.0\ MPa$ (E) *None*

10.( ) *The maximum shearing stress is*

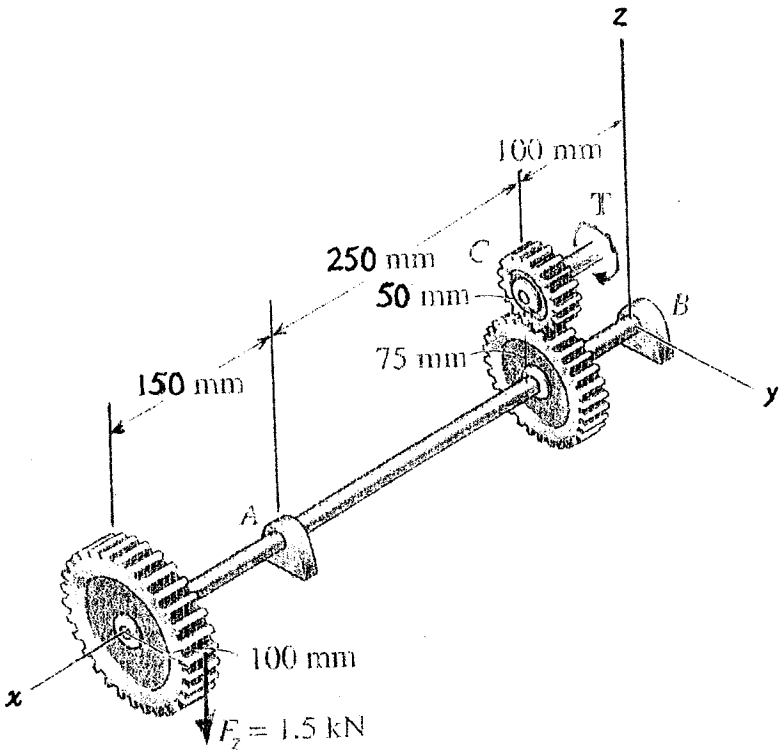
(A) $\sigma_{max} = 70.0\ MPa$ (B) $\sigma_{max} = 50.0\ MPa$ (C) $\sigma_{max} = 40.0\ MPa$
(D) $\sigma_{max} = 30.0\ MPa$ (E) *None*

Part II: To answer the following problems:

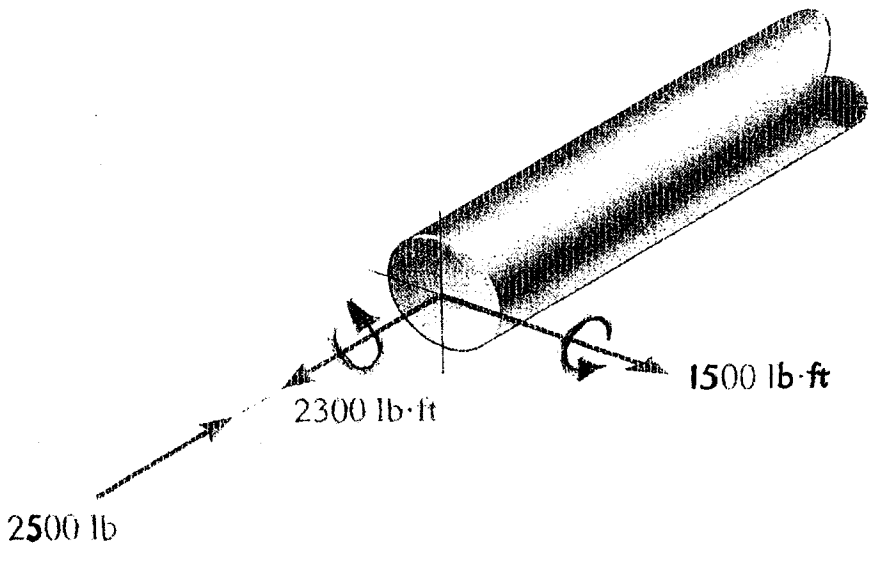
1. The fixed supported beam *AB* is strengthened using the simply supported beam *CD* and the roller at *F* which is set in place just before application of the load **P**. Determine the reactions at the supports if EI is constant. (20%)



2. The end gear connected to the shaft is subjected to the loading shown. If the bearings at *A* and *B* exert only *y* and *z* components of force on the shaft, determine the equilibrium torque *T* at gear *C* and then determine the smallest diameter of the shaft to the nearest millimeter that will support the loading. Using the maximum-shear-stress theory of failure with $\tau_{allow} = 60\ MPa$. (10%)



3. The internal loadings at a critical section along the steel drive shaft of a ship are calculated to be a torque of *2300 lb-ft*, a bending moment of *1500 lb-ft*, and an axial thrust of *2500 lb*. If the yield points of tension and shear are $\sigma_Y = 100\ ksi$ and $\tau_y = 50\ ksi$, respectively, determine the required diameter of the shaft using the maximum-distortion-energy theory. (20%)



1. (10%) Consider the system shown in Fig. 1 if $b > a > 0$,
   (a) (5%) please determine the breakaway point on the root locus,
   (b) (5%) please determine the points and gain value where the root loci cross the imaginary axis.
2. (20%) Consider the servo system shown in Fig. 2.
   (a) (4%) Please find the values of K_1 and K_2 so that the closed-loop poles are located at $-\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$.
   (b) (6%) From (a), please calculate the rising time t_r and maximum overshoot M_p when the system is only subjected to a reference input (unit-step input).
   (c) (10%) Obtain the steady-state error when the system is subjected to a reference input (unit-ramp input) and disturbance input (unit-step input)

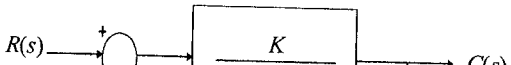


Fig. 1

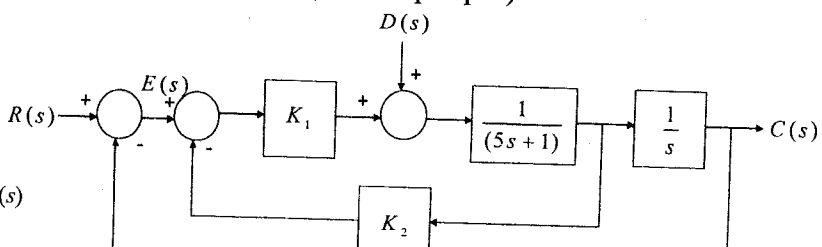


Fig. 2

3. (20%) The equivalent circuit shown in Fig. 3 is used to represent a PM dc motor and the motor variables and parameters are defined as follows.
$i_a(t)$: armature current; R_a: armature resistance; $e_b(t)$: back emf; T_L: load torque; T_m: motor torque; θ_m: rotor displacement; K_i: torque constant; L_a: armature inductance; $e_a(t)$: applied voltage; K_b: back-emf constant; ϕ: magnetic flux in the air gap; $\omega_m(t)$: rotor angular velocity; J_m: rotor inertia; B_m: viscous friction coefficient. The torque developed by the motor is proportional to the armature current $T_m(t) = K_i i_a(t)$. Please derive the transfer functions of $\frac{\Theta_m(s)}{E_a(s)}$ and $\frac{\Theta_m(s)}{T_L(s)}$.

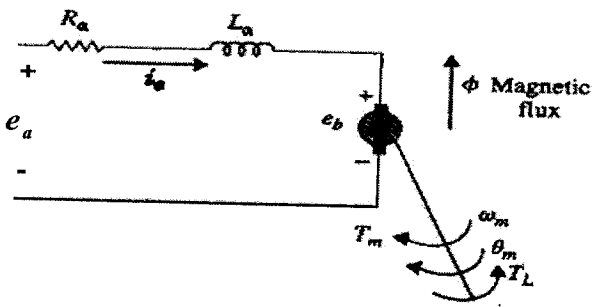
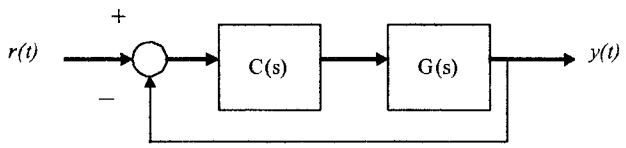


Fig. 3

4. (10%) Consider a linear dynamic system whose transfer function can be written as $\frac{20\cdot(s+10)}{s\cdot(s+200)}$. Please draw the polar plot of the system.

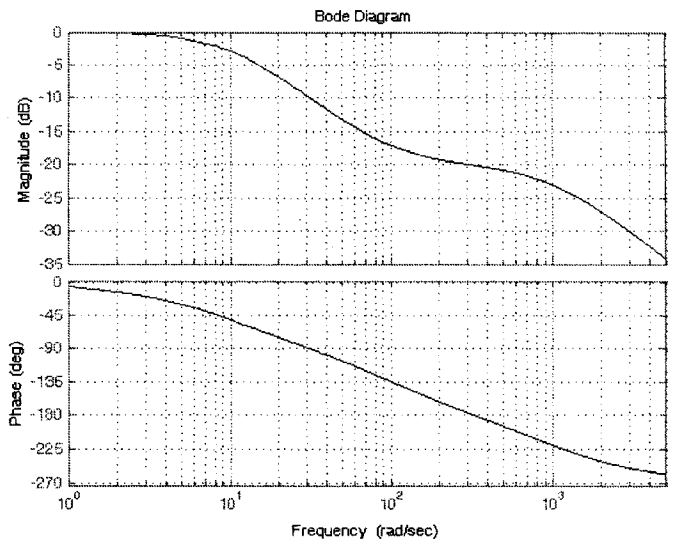
5. (20%) Consider a unit-feedback control system as shown below. The system output *y(t)* is required to follow a specific reference input *r(t)*, which is a periodic time function and can be written as $r(t)=\sum_{n=0}^{6} C_n\cdot\cos(n\cdot\omega_o t+\phi_n)$ with $\omega_o=10$.

Please draw a polar plot of the open-loop transfer function G(s)C(s) that may result in a good control performance. Please discuss the details of your polar plot as much as you can.

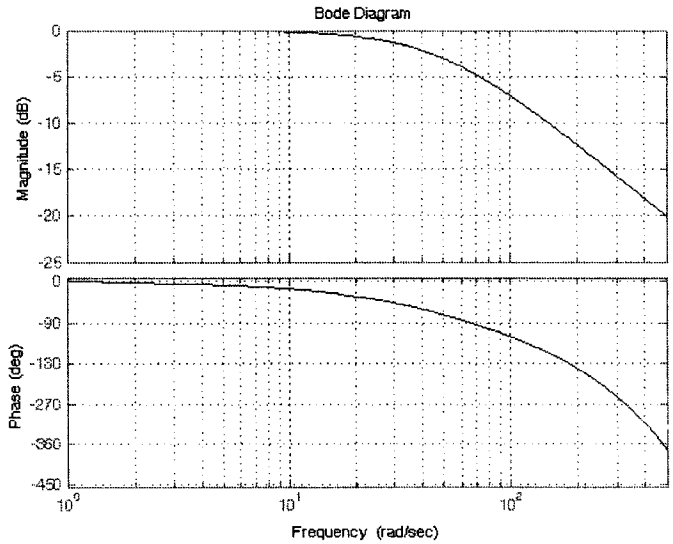


A unit feedback control system

6. (20%) It is given the Bode diagrams of two dynamic systems as indicated below. Please guess the possible transfer functions of the two systems.



(a)


Bode Diagram
Magnitude (dB)
0
-5
-10
-15
-20
-25
Phase (deg)
0
-90
-180
-270
-360
-450
10^0
10^1
10^2
Frequency (rad/sec)

(b)

一. 單一選擇題 60% (每題六分，共十題)

(1) What is the number of degrees of freedom of two particles connected by a rigid rod moving freely in a plane?
(A) 1, (B) 2, (C) 3, (D) 4, (E) 6

(2) A particle travels around a circle which a radius of 2 *m*. If the particle increases its speed at a constant rate of 6 m/sec^2 when its speed is 4 *m/sec*, what is the magnitude of acceleration of this particle at this instant?
(A) 8 m/s^2, (B) 10 m/s^2, (C) 12 m/s^2, (D) 14 m/s^2, (E) 16 m/s^2

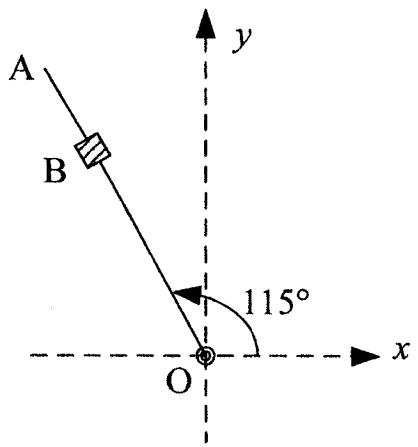
(3) A harmonic motion has an amplitude of 0.05 *m* and a frequency of 10 Hz. The period is:
(A) 0.05 *s*, (B) 0.1 *s*, (C) 0.2 *s*, (D) 0.5 *s*, (E) 1 *s*

(4) In Prob. (3), the maximum acceleration is:
(A) 39.48 m/s^2, (B) 78.96 m/s^2, (C) 157.91 m/s^2 (D) 197.39 m/s^2, (E) 394.79 m/s^2

(5) An industrial press is mounted on a rubber pad to isolate it from its foundation. If the rubber pad is compressed 5 *mm* by the self-weight of the press, the natural frequency of the system is about:
(a) 3 Hz; (b) 4 Hz; (c) 5 Hz; (d) 6Hz; (e) 7 Hz.

(6) A particle moves along a space curve governed by
$x = 6\cos\omega t$, $y = 4\sin\omega t$, $z = 3t^2$,
where *x*, *y*, and *z* are in meters, *t* is in seconds, and ω = 2 rad/sec. Compute the magnitude of its acceleration $\vec{a}$ when *t* = 4 seconds.
(A) 5.33 m/s^2, (B) 12.56 m/s^2, (C) 15.12 m/s^2, (D) 17.29 m/s^2, (E) 19.58 m/s^2

(7) A collar B slides along the 1-m arm OA in such a way that its distance from O is $r(t) = 1 - 0.25t^2$ m. The rotation of the arm OA about O is defined by $\theta(t) = 0.5t^2$ rad., where t is expressed in seconds. After the arm OA has rotated through 115°, determine the magnitudes of velocity (v) and acceleration (a) of the collar, respectively.
(A) v = 0.999 *m/s*, a = 4.03 m/s^2, (B) v = 1.999 *m/s*, a = 5.03 m/s^2, (C) v = 2.999 *m/s*, a =3.03 m/s^2, (D) v = 3.999 *m/s*, a = 2.03 m/s^2, (E) v = 4.999 *m/s*, a = 1.03 m/s^2



(8) Following the problem above, determine the magnitudes of the relative acceleration of collar with respect to the arm.
(A) 0.299 m/s^2, (B) 0.399 m/s^2, (C) 0.499 m/s^2, (D) 0.599 m/s^2 , (E) 0.699 m/s^2

(9) The following figure shows a mechanical system consisting of two masses on two springs. Derive the governing differential equations for the system.

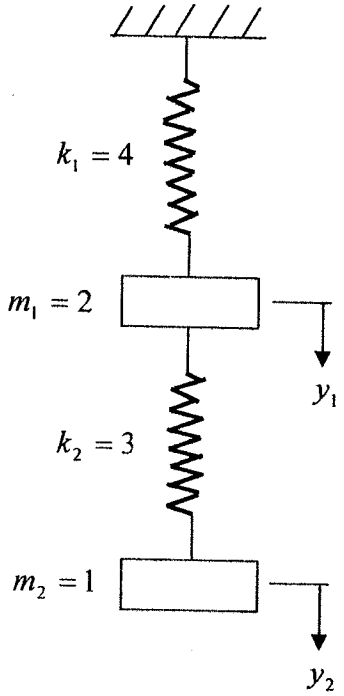
(A) $\begin{cases} \ddot{y}_1 + 7y_1 - 2y_2 = 0 \\ \ddot{y}_2 - 3y_1 + 3y_2 = 0 \end{cases}$

(B) $\begin{cases} 3\ddot{y}_1 + 6y_1 - 3y_2 = 0 \\ 2\ddot{y}_2 - 2y_1 + 3y_2 = 0 \end{cases}$

(C) $\begin{cases} 5\ddot{y}_1 + 3y_1 - 2y_2 = 0 \\ \ddot{y}_2 - 2y_1 + 7y_2 = 0 \end{cases}$

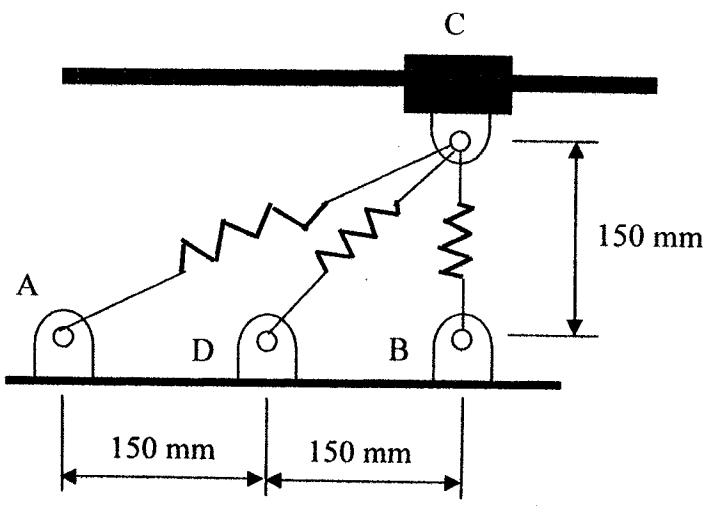
(D) $\begin{cases} 2\ddot{y}_1 + 7y_1 - 3y_2 = 0 \\ \ddot{y}_2 - 3y_1 + 3y_2 = 0 \end{cases}$

(E) $\begin{cases} 4\ddot{y}_1 + 7y_1 - 4y_2 = 0 \\ 3\ddot{y}_2 - 3y_1 + 7y_2 = 0 \end{cases}$



(10) A 1.2 *kg* collar C may slide without friction along a horizontal rod as shown below. About the three springs, spring constant k = 300 *N/m* and undeformed length is 150 *mm*. Given that the collar C is released from rest in the position shown. Determine the maximum velocity the collar C can reach.

(A) 0.76 *m/s*, (B) 1.76 *m/s*, (C) 2.76 *m/s*, (D) 3.76 *m/s*, (E) 4.76 *m/s*



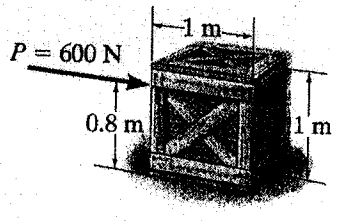
二. 簡答與計算題 (40%)

(1) Generally, in sequence, the analysis of a vibrating system involves four steps :
(a) _____, derivation of the governing equations,
(b)_____, (c) _____.
(6%)

(2) In dynamic systems, three common types of damping are (a) _____,
(b) _____, and Hysteretic damping. The logarithmic decrement represents the rate at which the amplitude of a free-damped vibration decreases. By measuring two consecutive displacement responses, the logarithmic decrement can be used to calculate the (c) _____ of a free-damped system.
(6%)

(3) An automobile is found to have a natural frequency of 20 *rad/s* without passengers and 17.32 *rad/s* with passengers of mass 500 *kg*. By treating the automobile as a single degree of freedom system, its mass is (a) _____ *kg*, and its stiffness is (b) _____ *N*/m. (8%)

(4) A uniform 50 *kg* crate rests on a horizontal surface for which the coefficient of kinetic friction is $\mu_k = 0.2$. A force of $P = 600$ *N* is applied to the crate as shown in the figure below.
(a) Draw the free body diagram for this crate. (6%)
(b) List the equations of motion. (Note: there are three equations of motion in this system.) (6%)
(c) Determine the crate's acceleration. (8%)



(a)

國立中山大學97學年度碩士班招生考試試題

科目：靜力學【機電系碩士班丁、戊組】

1. 設有一個剛體受到五個力的作用，下列之平衡方程式是依照該剛體的受力情況所列出，試分別就每一個情況繪出該剛體之受力情況的示意圖並加以簡要說明。(30%)

(a) $\sum_{i=1}^{5} F_{ix} = 0,\ \sum_{i=1}^{5} F_{iy} = 0$

(b) $\sum_{i=1}^{5} F_{ix} = 0,\ \sum_{i=1}^{5} F_{iy} = 0,\ \sum_{i=1}^{5} F_{iz} = 0$

(c) $\sum_{i=1}^{5} F_{iy} = 0,\ \sum_{i=1}^{5} M_{iz} = 0$

(d) $\sum_{i=1}^{5} F_{ix} = 0,\ \sum_{i=1}^{5} F_{iy} = 0,\ \sum_{i=1}^{5} M_{iz} = 0$

(e) $\sum_{i=1}^{5} F_{ix} = 0,\ \sum_{i=1}^{5} F_{iy} = 0,\ \sum_{i=1}^{5} F_{iz} = 0,\ \sum_{i=1}^{5} M_{iy} = 0,\ \sum_{i=1}^{5} M_{iz} = 0$

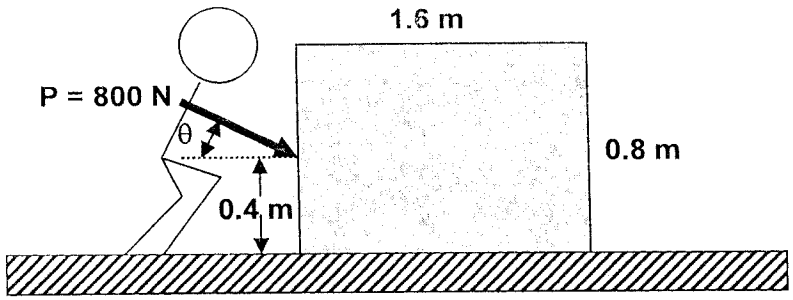
(f) $\sum_{i=1}^{5} F_{ix} = 0,\ \sum_{i=1}^{5} F_{iy} = 0,\ \sum_{i=1}^{5} M_{ix} = 0,\ \sum_{i=1}^{5} M_{iy} = 0,\ \sum_{i=1}^{5} M_{iz} = 0,$

2. 有一空心鐵櫃如圖重 100 Kg，一人以夾角 30° (θ=30°)施力 800 N 推該鐵櫃，假設 $\mu_s = 0.3, \mu_k = 0.25$。

(a) 請畫出該系統之自由體圖(Free-Body Diagram)。 (3%)

(b) 請寫出支平衡方程式(Equations of Equilibrium)。(3%)

(c) 決定該鐵櫃之移動速度。(4%)

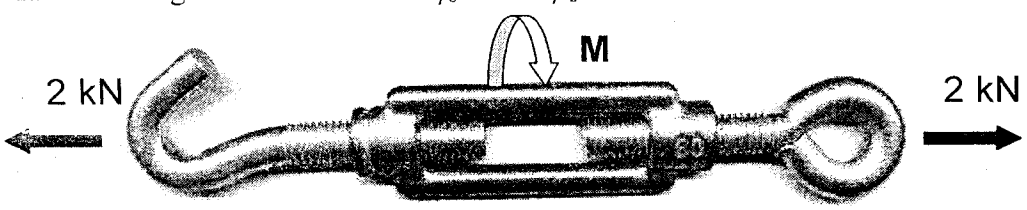


3. A turnbuckle shown as below has a square thread with a mean radius of 5 mm and a lead of 2 mm. Assume $\mu_s = 0.25$.

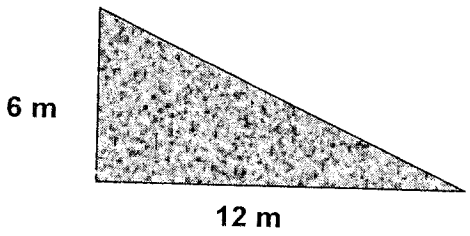
(a) Determine the moment M that must be applied to draw the end screws closer together. (5%)

(b) Determine if the turnbuckle is still self-locking when the moment M is removed? (5%)

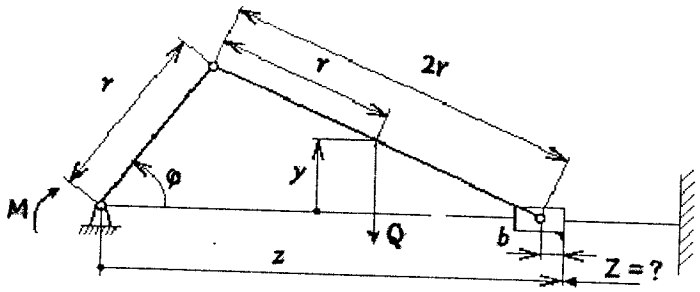
Hint: the angle of static friction $\phi_s = tan^{-1}\mu_s$.



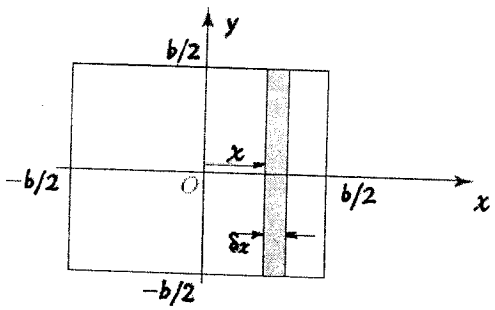
4. (a) 試求下圖三角型之形心(centroid) (5%)
(b) 請問質心(center of mass)和重心(center of gravity)的定義，以及在何種狀況下，同一剛體的質心及重心兩點不重合？（5%）



5. (a) What is "Principle of virtual work"? (5%)
(b)Please use Principle of virtual work to determine the magnitude of a force Z for equilibrium of a crank-slider mechanism in the position given by the angle $\varphi = 30^{\circ}$. Given is $M = 50$ Nm, $Q = 35$ N, $r = 0.1$ m. (15 %)



6. Find the moment of inertia of the square lamina below about one of its sides. (10%)



7. Find x_{eq} such that the equilibrium position of a particle located on the end of a massless bar is assured. Horizontal force $F = 63$ N, vertical force $G = 90$ N. The length of the bar is $l = 0.08$ m. (10 %)