國立中山大學九十一學年度碩士班招生考試試題

科目：工程數學（機電系碩士班甲乙丙丁組）

I. 選擇題部份（單選題）（每題 4 分，共 60 分）

1. Determine values of C and K so that the following equation is exact:

$$Cx^2ye^y + 2\cos(y) + \left(x^3e^y y + x^3e^y + Kx\sin(y)\right)y' = 0.$$

Which of the following are their values?

(A) $C = 2, K = 3$, (B) $C = 3, K = 2$, (C) $C = -2, K = 3$, (D) $C = 3, K = -2$, (E) $C = -2, K = -3$.

2. Evaluate $\oint_C |z|^2 dz$, C: $|z| = 2$, (counterclockwise). Which of the following is the value of the integral?

(A) 0, (B) πi, (C) $2\pi i$, (D) $4\pi i$, (E) 2.

3. Which of the following statement is always true?

(A) The eigenvalues of a Hermitian matrix have absolute value 1.

(B) The eigenvalues of a skew-Hermitian matrix have absolute value 1.

(C) The eigenvalues of a unitary matrix are real.

(D) The eigenvalues of a Hermitian matrix are real.

(E) The eigenvalues of a unitary matrix are zero or pure imaginary.

4. $f(x)$ is a periodic function of period 2π that can be represented by a trigonometric series,

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx).$$

Let $f(x) = \begin{cases} 0 & \text{if } 0 < x < \pi \\ 1 & \text{if } \pi < x < 2\pi \end{cases}$. Which of the following is the value of a_0?

(A) 0, (B) 0.5, (C) 1.0, (D) 1.5, (E) 2.0

5. Using the Fourier series of the given function given in Prob. 4, find the sum of $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + - \cdots$. Which the following is the sum?

(A) $\pi/4$, (B) $\pi^2/12$, (C) $\pi^3/40$, (D) $\pi^4/120$, (E) none of the above.

6. Consider a group of *30* employees, *20* of whom possess M.S. degree, what is the probability of selecting without replacement 3 persons who hold M.S. degrees?

(A) 0.12 (B) 0.18 (C) 0.22 (D) 0.28 (E) 0.33

7. The forced response of the ODE $(D^2 + 3D + 2)y(t) = Df(t)$ where $f(t) = 10e^{-3t}$ and $y(0) = 0$; $dy(0)/dt = 5$. is

(A) $-15e^{-3t}$ (B) $-10\,e^{-t}$ (C) $25e^{2t}$ (D) $-10\,e^{t}$ (E) $3e^{-3t}$

8. If $\mathbf{r}(t) = 5\cos t\,\mathbf{i} + 5\sin t\,\mathbf{j}$ is the position vector of a moving particle, then the normal components of the acceleration at any t is

(A) 0 (B) 0.5 (C) 5 (D) 10 (E) 15

9. To find the steady state temperature distribution of a circular plate, one needs to solve

(A) a wave equation in polar coordinates

(B) a heat equation in polar coordinates

(C) a Laplace equation in polar coordinates
(D) a wave equation in spherical coordinates
(E) a heat equation in spherical coordinates

10. What is $\partial z/\partial v$ if $z = 4x\text{-}5y^2$, $x = u^4 - 8v^3$ and $y = (2u - v)^2$?

(A) $-96v^2 + 20y(2u - y)$ (B) $16u^3 - 40y(2u - v)$ (C) $x^2 - 2y\,(v - u)$
(D) $24u^4 - 2y(3v - 3)$ (E) $-12v + 24x(u - v)$

11. The particular solution of the fourth order differential equations $\frac{d^4y}{dx^4} - y = 4.5\,e^{-2x}$ is

(A) $4.5\,e^{-2x}$ (B) $0.3\,e^{-2x}$ (C) $4.5e^{x} + 0.3e^{-x}$ (D) $4.5\cos x + 0.3\sin x$ (E) None

12. The Laplace transform of the function $f(t) = e^{-t}\sin 2\pi t$ is

(A) $\frac{2\pi}{(s+1)^2 + 4\pi^2}$ (B) $\frac{s^2 - 2}{s^3 - 4s}$ (C) $\frac{s^2 - 2}{(s^2 + 1)s^3 - 4s}$ (D) $\frac{1}{2s} + \frac{s}{2s^2 - 2\pi}$

(E) None

13. Consider $z = x + iy$ and $\bar{z}$ be the complex and complex conjugate pair variable, then which of the followi complex function is not analytic?

(A) $f(z) = z^8$ (B) $f(z) = \frac{1}{1 - z}$ *and* $z > 2$ (C) $f(z) = e^{x}(\cos y + i\sin y)$

(D) $f(z) = z^2 - \bar{z}^2$ (E) None

14. The residue at the singular point of the complex function $f(z) = \frac{3}{1 - z}$ is

(A) -3 (B) $\frac{-1}{3i}$ (C) $\frac{i}{3}$ (D) 1 (E) None

15. Which equation is nonlinear?
(A) $y''' - y'' - 4y' + 4y = 6e^{-x}$ (B) $y'' + 4y = 10\cos x$ (C) $y'' + xy' = 4x$
(D) $y''' + 3y' - 4y = \sin x$ (E) None

II. 計算題部分 (三題共計 40 分)

1. Find the general solution of following equations (15%)

$$\begin{cases} y_1' = -y_1 + y_2 \\ y_2' = -y_1 - y_2 \end{cases}$$

2. The temperature at a point (x, y) on a flat surface is given by $T(x, y) = 100 - 2x^2 - y^2$. Find the path a heat-seeking robot will take, starting at (3, 4), as it moves in the direction in which the temperature increases most rapidly. (15%)

3. Using Laplace Transform method, solve the following initial value problem

$$y'' - 2y' + y = e^{t},\quad y(0) = 1 \quad and \quad y'(0) = 0 \qquad (10\%)$$

1. Consider a rigid and evacuated container (bottle) of volume V that is surrounded by the atmosphere (T_o, P_o). At some point in time, the neck valve of the bottle opens, and atmospheric air gradually flows in. The wall of the bottle is thin and conductive enough so that the trapped air and the atmosphere eventually reach thermal equilibrium. In the end, the trapped air and the atmosphere are also in mechanical equilibrium, because the neck valve remains open. Please determine the net heat transfer interaction that take place through the wall of the bottle during the entire filling process.(15%)

2. Please derive the first law efficiencies between two thermal reservoirs T and T_o for idea heat engines, refrigerators, and heat pumps. Then make a plot to explain the ranges of the first law cycle efficiencies of the three types of devices. In the plot, please display the upper bound of the first law efficiencies obtained from the assumptions of the above idea models. In heat engine and heat pump, the high temperature thermal reservoir is denoted as T, and the low temperature thermal reservoir is denoted as T_o. However, in the refrigerator, the high temperature thermal reservoir is denoted as T_o and the low temperature thermal reservoir is denoted as T. In this plot, the ordinate could be thermal efficiency of heat engine, the coefficient of performance of refrigerator, or the coefficient of performance of heat pump, and the abscissa is T/T_o.(15%)

3. To determine the specific volume of superheated water at a known pressure and temperature, when would you use each of the following: the steam tables, the generalized compressibility chart, an equation of state, the idea gas model?(5%)

4. The Joule-Thomson coefficient can be written

$$\mu_J = \frac{T^2}{c_p}\left(\frac{\partial(v/T)}{\partial T}\right)_p$$

(a) Using this relation, obtain an expression for the Joule-Thomson coefficient for a gas obeying the equation of state

$$v = \frac{RT}{p} - \frac{Ap}{T^2}$$

where A is a constant.

(b) Using the result of part (a), determine c_p, in kJ/kg · K, for CO_2 at 400 K, 1 atm, where μ_J=0.57 K/atm. For CO_2, $A=2.78\times10^{-3}$ m^5 K^2/kg · N.

(15%)

5. **Write down two statements of the second law of thermodynamics and prove that they are equivalent to each other.** (15%)

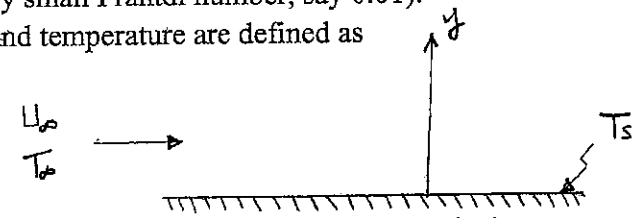
6. Draw P-v and T-s diagrams for each of the following ideal cycles:
   (a) Carnot cycle (4%)
   (b) Stirling cycle (4%)
   (c) Brayton cycle (4%)
   (d) Rankine cycle (4%)
   (e) Otto cycle (4%)

7. A circular ice rink is 20 m in diameter and is to be temporarily enclosed in a hemispherical dome of the same diameter. The ice is maintained at 270 K, and on a particular day the inner surface of the dome is measured to be 290 K. Estimate the radiant heat transfer from the dome to the rink if both surfaces can be taken as black. (15%)

1. (9%) Give the definition of Prantdl number and its physical meaning. Sketch the boundary layer profiles of dimensionless velocity and temperature for a liquid metal flowing horizontally over a cold flat plat (The liquid metal has very small Prantdl number, say 0.01). Dimensionless velocity and temperature are defined as

$$\bar{u}(y) = \frac{u(y)}{U_\infty}$$

$$\bar{T}(y) = \frac{T(y) - T_s}{T_\infty - T_s}$$



where U_∞ and T_∞ are free stream velocity and temperature. T_s is the surface temperature.

2. (10%) Experimental measurements of the temperature distribution during the flow of air at atmospheric pressure over the wing of an airplane indicate that the temperature distribution near the surface can be approximate by a linear equation:

$(T-T_s)=ay(T_\infty-T_s)$

where a= a constant =2 m^{-1}, T_s =surface temperature, K, T_∞=free-stream temperature, K, and y=perpendicular distance from surface (mm)

(a) Estimate the convection heat transfer coefficient if T_s =50° C and T_∞=-50° C.

(b) Calculate the heat flux in W/m^2.

(Note: Some data of air are provided in Table 1)

3. (16%) A vertical cylinder of 0.01m diameter and 0.3 long has a surface temperature of 400K. If it is immersed in air at 350K, natural convection will occur as a result of the temperature difference. We want to know the heat transfer rate.

(a) Calculate the Grashof and Rayleigh numbers for the natural convection problem

(b) Determine the Nusselt numbers and the heat transfer coefficient

(c) Calculate the heat transfer rate due to the natural convection

(d) If the air has a velocity 2m/s, approaching the cylinder horizontally, should the forced convection be considered in the problem? Explain your answer.

(Note: Table 1 and Figure 1 are provided in the following for your reference)

4. (5%)Define a Newtonian fluid.

5. (10%)Why a fluid can be treated as a continuum?

6. (10%)What is the Reynolds transport theorem (RTT)? Write down the conservation of mass and the conservation of momentum in Eulerian form using RTT.

7. (10%)Define a problem of general Couette flow.

8. (10%)For the same U_∞ flow over a ball, compare the shear drag for turbulent and laminar flows. Also compare the total drag in two flows.

9. (10%)Write down the Prandtl boundary layer equation. Suggest how to solve it.

10. (10%)What is the boundary layer separation? When will the boundary layer separation occur? What is the adverse pressure gradient? What is the favorable pressure gradient?

（接下頁）

Table 1

TABLE | Dry Air at Atmospheric Pressure

Temperature, T			Density, ρ (kg/m^3)	Coefficient of Thermal Expansion, $\beta \times 10^3$ (1/K)	Specific Heat, c_p (J/kg K)	Thermal Conductivity, k (W/m K)	Thermal Diffusivity, $\alpha \times 10^6$ (m^2/s)	Absolute Viscosity, $\mu \times 10^6$ (N s/m^2)	Kinematic Viscosity, $\nu \times 10^6$ (m^2/s)	Prandtl Number, Pr	$\frac{g\beta}{\nu^2} \times 10^{-8}$ (1/K m^3)
°F	K	°C	$\times 6.243 \times 10^{-2}$ = (lb_m/ft^3)	$\times 0.5556$ = (1/R)	$\times 2.388 \times 10^{-4}$ = (Btu/lb_m °F)	$\times 0.5777$ = (Btu/hr ft °F)	$\times 3.874 \times 10^4$ = (ft^2/hr)	$\times 0.6720$ = (lb_m/ft s)	$\times 3.874 \times 10^4$ = (ft^2/hr)		$\times 1.573 \times 10^{-1}$ = (1/R ft^3)
32	273	0	1.252	3.66	1011	0.0237	19.2	17.456	13.9	0.71	1.85
68	293	20	1.164	3.41	1012	0.0251	22.0	18.240	15.7	0.71	1.36
104	313	40	1.092	3.19	1014	0.0265	24.8	19.123	17.6	0.71	1.01
140	333	60	1.025	3.00	1017	0.0279	27.6	19.907	19.4	0.71	0.782
176	353	80	0.968	2.83	1019	0.0293	30.6	20.790	21.5	0.71	0.600
212	373	100	0.916	2.68	1022	0.0307	33.6	21.673	23.6	0.71	0.472
392	473	200	0.723	2.11	1035	0.0370	49.7	25.693	35.5	0.71	0.164
572	573	300	0.596	1.75	1047	0.0429	68.9	29.322	49.2	0.71	0.0709
752	673	400	0.508	1.49	1059	0.0485	89.4	32.754	64.6	0.72	0.0350
932	773	500	0.442	1.29	1076	0.0540	113.2	35.794	81.0	0.72	0.0193
1832	1273	1000	0.268	0.79	1139	0.0762	240	48.445	181	0.74	0.00236

Source: K. Raznjević, *Handbook of Thermodynamic Tables and Charts*, McGraw-Hill, New York, 1976.

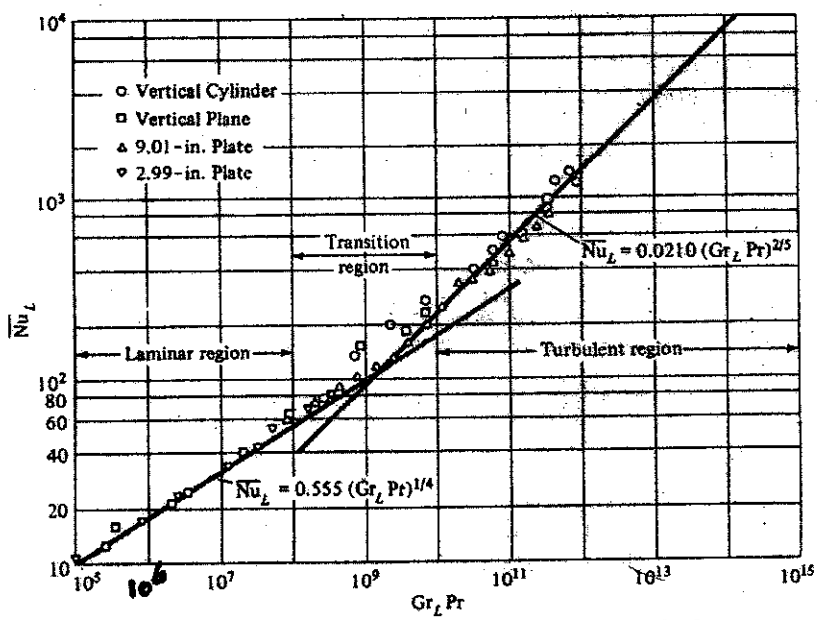
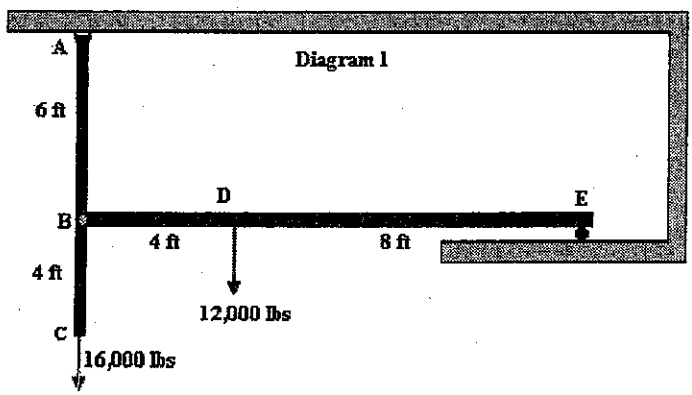


FIGURE | Correlation of Data for Natural-Convection Heat Transfer from Vertical Plates and Cylinders (7).

[I] Please chose the correct answers for following problems
Please be noted that the answers for each problem may be more than one

(1) In the structure shown in Diagram 1, member BDE is supported by a roller at point E, and is pinned to member ABC at point B. Member ABC is pinned to the wall at point A. Member ABC is an aluminum rod with a diameter of 1 inch. (Young's Modulus for Aluminum is 10 x 10^6 lb/in^2)



Which statements are correct? (10%)
(A) The member ABC is an axial member.
(B) The normal stress in section AB equals to the normal stress in section BC.
(C) 20,000psi < (the normal stress in section BC) < 21,000psi.
(D) 0.29 inches < (the movement of point C) < 0.34 inches.
(E) None of the previous statements is correct.

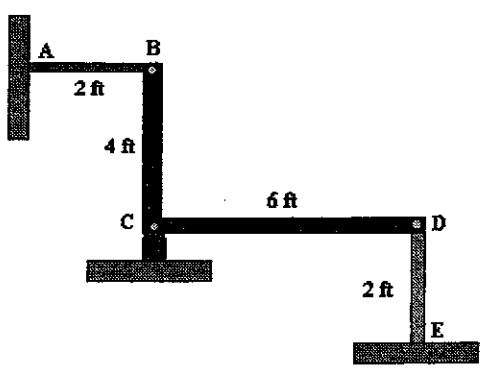
(2) In the structure shown the solid rigid L-shaped member BCD is supported by Steel rod AB and Aluminum member DE, and pinned at point C, as shown. Member DE has a cross sectional area of 1 in^2 and member AB have a cross sectional area of 0 .5 in^2. The structure is initially unstressed and then experiences a temperature decrease of 60 degrees Celsius.
The following values may be needed for the problem below

Linear coefficient of Expansion:

$\alpha_{Steel} = 6.5 \times 10^{-6}/°F$; $\alpha_{Aluminum} = 13 \times 10^{-6}/°F$;

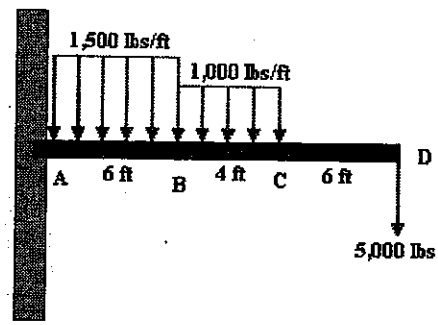
Young's Modulus:

$E_{Steel} = 30 \times 10^6$ lb/in^2; $E_{Aluminum} = 10 \times 10^6$ lb/in^2;



Which statements are correct? (10%)
(A) Both the steel member (AB) and the aluminum member (DE) would like to contract.
(B) The resulting movement of point D is greater than the resulting movement of point B.
(C) 28,000psi < (the stress in steel member AB) < 32,000psi.
(D) (The axial force in member DE) = 1.5 (the axial force in member AB)
(E) None of the previous statements is correct.

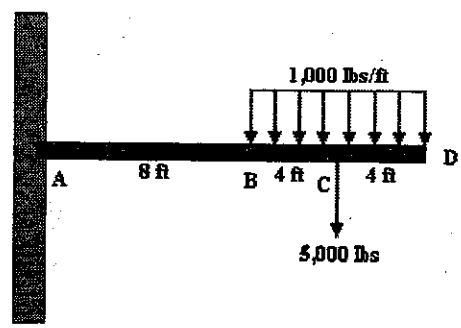
(3) A loaded, cantilever beam is shown below.



For this beam, which statements are correct? (10%)

(A) The shear force for section AB is a linear function.
(B) The shear force for section CD is a constant.
(C) The bending moment for section BC is a linear function.
(D) The bending moment for section CD is a constant.
(E) None of the previous statements is correct.

(4) A loaded, cantilever 2 inches x 10 inches rectangular beam is shown below:



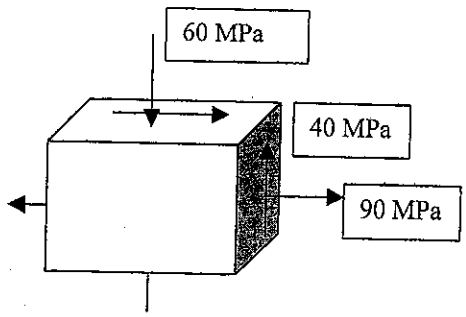
For this beam, which statements are correct? (10%)

(A) The magnitude of the shear force at a point 4 feet from the left end of the beam is 13,000 lbs.
(B) The magnitude of the bending moment at a point 4 feet from the left end of the beam is 104,000 ft-lbs.
(C) 35,000 psi <(the magnitude of the maximum bending stress at a point 4 feet from the left end of the beam) < 40,000 psi.
(D) 800 psi < (the horizontal shear stress at a point 6 inches above the bottom of the beam cross section and 4 feet from the left end of the beam.) < 1,200 psi.
(E) None of the previous statements is correct.

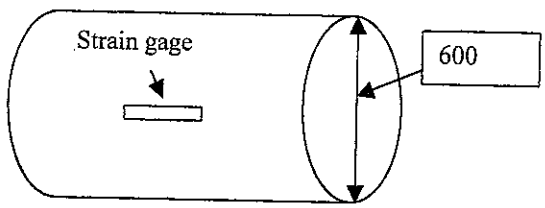
(5) In the topic of column buckling, the "slenderness ratio" is defined as the (effective) length of the column divided by its radius of gyration. The effective length , L_e, is given by: $L_e = K L$, where **L** is the original length of the column and **K** could be called an effective length constant. The values for **K** depend on how the column is supported. Which statements are correct? (10%)

(A) If both the ends of the column are pinned, then **K** = 1.
(B) If both the ends of the column are fixed, then **K** = 1.
(C) If one end of the column is pinned and the other end is fixed, then **K** = 2.
(D) If one end of the column is fixed and the other end is free, then **K** = 2.
(E) None of the previous statements is correct.

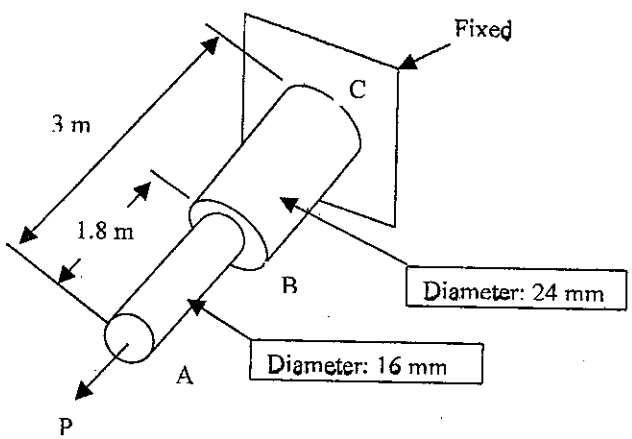
[II]. (15%) For the state of plane stress shown below, determine (a) the principal planes, (b) the principal stresses, (c) the maximum shearing stress.



[III]. (20%) A strain gage is attached horizontally to the cylindrical surface of a pressure vessel of 600 mm outside diameter and 7.5 mm wall thickness. Knowing that E=200 GPa and ν =0.25 and that the strain gage reads 120 μ, determine (a) the three principal strains on the cylindrical surface of the vessel, (b) the principal stresses in the wall, (c) the gage pressure inside the vessel.



[IV] (15%) Using E=75 GPa, (i) if P=60kN, determine (a) the strain energy of the aluminum rod ABC, (b) the corresponding strain-energy density in portions AB and BC of the rod. (ii) If σ_Y = 400 MPa, determine the maximum load P without causing any permanent deformation in ABC.

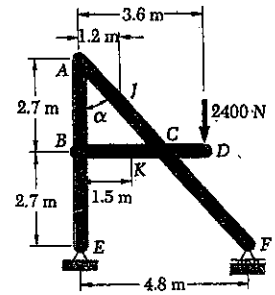


國立中山大學九十一學年度碩士班招生考試試題

科目：應用力學(靜力學與動力學)（機電系乙丙組）共 1 頁 第 1 頁

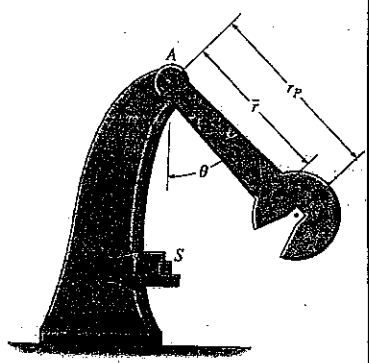
(Make assumptions which you think are needed.)

1. In the frame shown, determine the internal forces at point J of member ACF. (12%)



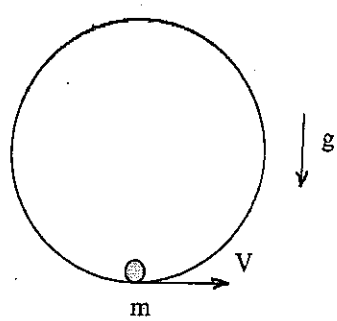
2. For a rectangular plate in a two-dimensional plane, give one example for each of the following 4 cases: (a). complete fixity with minimum number of adequate constraints, (b). partial fixity with inadequate constraints, (c). complete fixity with redundant constraints, (d). partial fixity with redundant constraints. (8%)

3. The Charpy impact test is used in materials testing to determine the energy absorption characteristics of a material during impact. The test is performed using the pendulum shown. The pendulum has a mass m, mass center at G, and a radius of gyration k_G about G. Determine the distance r_P from the pin at A to the point P where the impact with the specimen S should occur so that the horizontal force at the pin is essentially zero during the impact. For the calculation, assume the specimen absorbs all the pendulum's kinetic energy gained during the time it falls and thereby stops the pendulum from swinging when q = 0°. (20%)



4. Derive the principle of work and energy and the principle of linear impulse and momentum for a particle by Newton's second law $\sum \bar{F} = m\bar{a}$. (20%)

5. A particle of mass m moves along the inside of a circular hoop (radius r) without friction as shown. Please determine the minimum speed V (horizontal) of the particle so that it will maintain contact with the track surface all the time. (20%)



6. A cart of mass M was moving with constant speed V on a flat horizontal surface without friction. What would be the speed of the cart if an amount of water of mass m was poured vertically down into it? Later, the same amount of water drained away through a hole underneath the cart, what would be the speed of the cart then? (20%)

國立中山大學九十一學年度碩士班招生考試試題

科目：自動控制（機電系）（丙組）

一、(25%) 試述增加零點與極點對控制系統的影響。請就對於時域、頻域響應、根軌跡等的影響分別敘述之。如果要設計一個控制器，你要如何決定要增加零點或極點或二者都要？零點或極點的位置該如何決定？

二、(1) (20%) 試述繪製根軌跡圖的方法，並詳細說明(或證明)這些方法的理論依據。

(2)(5%) 為何在根軌跡的分離點(breakaway point) $\frac{dK}{ds}=0$.

三、考慮一單位回授(unity feedback)之非極小相位(non-minimum phase)控制系統，已知其開迴路轉移函數(open-loop transfer function)為

$$\frac{k(z_0-s)\prod_{j=1}^{n-2}(s+z_j)}{s\prod_{i=1}^{n}(s+p_i)}$$

其中 k, z_0, z_j, 與 p_i 均為正實數。

(1)(15%) 當給定一步級函數(step function)命令時，**證明**其步級響應將會先往與命令之相反方向進行。

(2)(10%) 為使斜坡輸入(ramp input)之穩態誤差(steady-state error)能達到 e_r，則 k 值應如何設計？

四、一般直流馬達可簡單以一階低通濾波器(low-pass filter)的形式描述其動態行為（輸入為電壓，輸出為轉速）。已知一直流馬達之頻寬為 ω_{BW} (rad/sec)。

(1)(10%) 在接受一恆定電壓 5 V，馬達從靜止開始轉動，穩態速度為 10 rad/sec，其安定時間(settling time)約是多少？

(2)(5%) 給定一弦波(sinusoidal)電壓，如其頻率為馬達頻寬之**十分之一**，繪圖表示輸入電壓與馬達速度響應之關係。

(3)(10%) 如弦波電壓頻率為頻寬之**十倍**，繪圖表示輸入電壓與馬達速度響應之關係。

1. For the slider-crank mechanism given as shown in Fig. 1. Please determine: (a) the displacement, (b) the velocity, and (c) the acceleration at pivot B as functions of link length a, b, and the crank position θ. (25%)

2. A particle is sliding out with a constant relative velocity ν, relative to the slot, on a disk while the disk is rotating at a constant angular velocity ω as shown in Figure 2. Determine: (a) the velocity and (b) the acceleration of the particle A as functions of r, ν, θ, and ω. (25%)

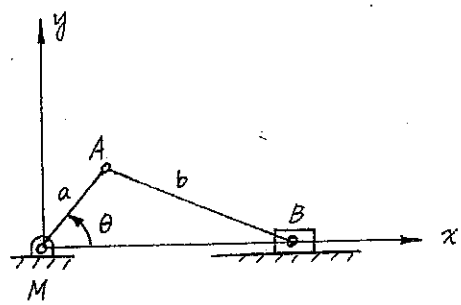


Figure 1

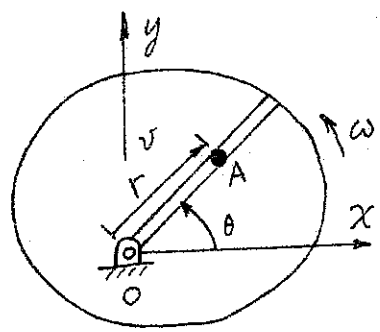


Figure 2

3. In Fig. 3 the motor at C pulls in the cable with an acceleration $a_C = (3t^2)$ m/s^2, where t is in seconds. The motor at D draws in its cable at $a_D = 5$ m/s^2. If both motors start at the same instant from rest when $d = 3$ m, determine (a) the time needed for $d = 0$, and (b) the relative velocity of block A with respect to block B when this occurs. (25%)

4. The 50-lb wheel shown in Fig. 4 has a radius of gyration $k_G = 0.70$ ft. If a 35-lb-ft couple moment is applied to the wheel, determine the acceleration of its mass center G. The coefficients of static and kinetic friction between the wheel and the plane at A are $\mu_s = 0.3$ and $\mu_k = 0.25$, respectively. (25%)

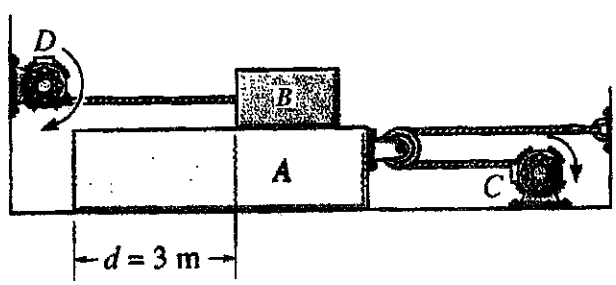


Figure 3.

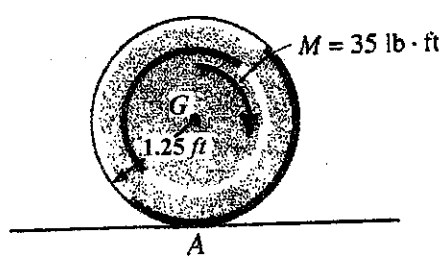


Figure 4.

國立中山大學九十一學年度碩士班招生考試試題

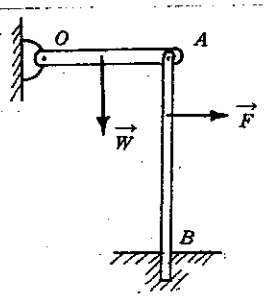
科目：靜力學（機電系）(丁組)

一、如圖一所示，試畫出桿 OA 與桿 OB 的自由體圖(Free-body diagram)。(8%)

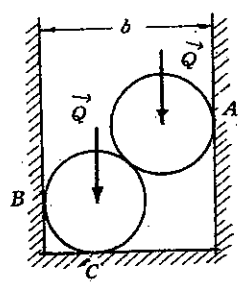
二、如圖二所示，兩個光滑圓球，半徑皆為 10 公分，重量皆為 50 公斤，它們靜止在一個光滑的槽內，槽的寬度為 36 公分，試求圓球在 A，B，與 C 三個接觸點處所受的力。(12%)

三、如圖三所示，AB 之桿長為ℓ，重量皆為 W，端點 A 以光滑鉸鏈予以固定；端點 B 與一平板接觸，平板則置於水平地面上，其重量不計。設兩個接觸面的磨擦係數皆為μ。若不論平板所受的拉力 P 的大小如何變化，欲使平板不會發生滑動，則試求μ應有之值。 (15%)

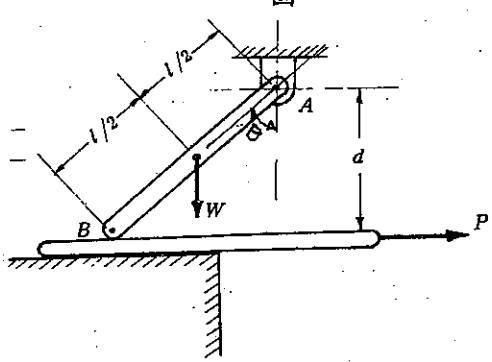
四、如圖四所示，ABCD 為一平面之單構架，在接頭 B 上受有一外力 P，其作用線與 BC 桿之中心線重合。試求各桿件因外力 P 所產生的軸向力。(15%)



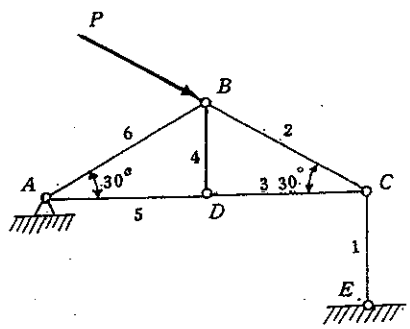
圖一



圖二



圖三



圖四

五、在平面上有力 **F** 並 **F** 可分解為 F_1 與 F_2 兩分力。試證明 ***Varignon's Theorem,*** 即力 **F** 對固定點 *O* 所取的力矩會等於其分力 F_1 與 F_2 對點 *O* 所取的力矩和。(15%)

六、什麼是 Virtual work? 舉例說明如何將其應用在靜力的分析上。(15%)

七、在你做過的許多靜力題目當中，有哪些是屬於線性的系統而又有哪些是屬於非線性的系統？試以 Superposition 方法的可適用性來說明此二類系統的差異。(20%)