國立中山大學九十二學年度碩士班招生考試試題

科目： 工程數學 (機械與機電工程學系碩士班 甲.乙.丙.丁組)

I. 選擇題部分(單選題) (每題 5 分，共 60 分)

1. The residue at the singular point of the complex function $f(z)=\frac{z^2+2}{1-z}$ is (A) 3 (B) −3 (C) 1 (D) −1 (E) None.

2. Let C be the circle $|z|=2$, described in the positive sense. The integral $\frac{1}{\pi i}\oint_C \tan z dz$ is (A) 2 (B) −2 (C) 4 (D) −4 (E) None.

3. If $\cos z = 2$, then $\cos 3z$ is (A) 7 (B) 19 (C) 26 (D) 45 (E) None.

4. Which of the following is the solution of $x^2y''+xy'+(x^2-0.25)y=0$?

   (A) $\sin x$ (B) $x\sin x$ (C) $\sin x/\sqrt{x}$ (D) $\sqrt{x}\sin x$ (E) $(\sin x)/x$.

5. The differential equation $x^2y''+2xy'+\lambda^2x^2y=0$ can be transformed into the Bessel equation: $\xi^2 z''+\xi z'+(\xi^2-\nu^2)z=0$ by setting $y(x)=z(\xi)/\sqrt{x}$, $\xi=\lambda x$. What is the value of ν? (A) 0 (B) 0.5 (C) 1 (D) 2 (E) None.

6. Evaluate the improper integral of $\int_0^\infty \frac{dx}{x^2+1}$. Which of the following is the value of the integral? (A) π/6 (B) π/4 (C) π/2 (D) π (E) None.

7. Evaluate the improper integral of $\int_0^\infty \frac{dx}{(x^2+1)^2}$. Which of the following is the value of the integral? (A) π/6 (B) π/4 (C) π/2 (D) π (E) None.

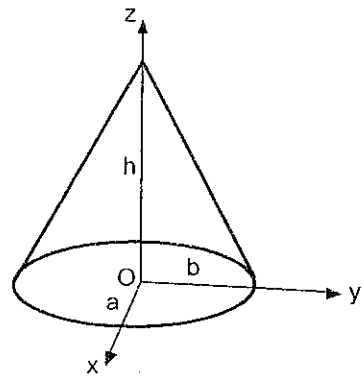
8. Selection of the "best" members from a finite set of candidates is a very important problem for many companies and represents a serious challenge for many personnel directors. Now, without very little knowledge of how to make a selection from a group of 20 applicants, a personnel director decides to randomly select ten applicants for employment. What is the probability that the 10 selected include all the 5 best applicants in the group of 20? (A) 0.0163 (B) 0.0136 (C) 0.0234 (D) 0.1422 (E) 0.0033.

9. This problem considers the functions $f(t) = t$ and $x(t) = 1$ for $0 \leqq t \leqq 1$ and $f(t) = x(t) = 0$ for $t > 1$ and $t < 0$. Find the component of the form $x(t)$ contained in $f(t)$. In other words, find the optimum value of c for $cx(t)$ so that the error between $f(t)$ and $cx(t)$ can be minimized. By noting that the error is defined as the integral of $(f(t) - cx(t))^2$, what is the optimum value of c? (A) 0.75 (B) 0.25 (C) 0.33 (D) 0.50 (E) 0.60.

題目 10-12

The following figure is a solid elliptic cone. The center of the bottom surface coincides with the origin of the reference coordinate system.

10. The volume of the solid elliptic cone is (A) $\frac{\pi abh}{4}$, (B) $\frac{\pi abh}{3}$, (C) $\frac{\pi abh}{2}$, (D) $\frac{2\pi abh}{3}$, (E) other than above.



11. The area of the cross-sectional ellipse at $z = h/2$ is (A) $\frac{\pi ab}{4}$, (B) $\frac{\pi ab}{3}$, (C) $\frac{\pi ab}{2}$, (D) $\frac{2\pi ab}{3}$, (E) other than above.

12. The area of the lateral surface on the solid elliptic cone is

(A) $\frac{1}{4}\int_0^{2\pi}\sqrt{(h^2+a^2\cos^2\theta+b^2\sin^2\theta)(a^2\cos^2\theta+b^2\sin^2\theta)}d\theta$,

(B) $\frac{1}{3}\int_0^{2\pi}\sqrt{(h^2+a^2\cos^2\theta+b^2\sin^2\theta)(a^2\cos^2\theta+b^2\sin^2\theta)}d\theta$,

(C) $\frac{1}{2}\int_0^{2\pi}\sqrt{(h^2+a^2\cos^2\theta+b^2\sin^2\theta)(a^2\cos^2\theta+b^2\sin^2\theta)}d\theta$,

(D) $\frac{2}{3}\int_0^{2\pi}\sqrt{(h^2+a^2\cos^2\theta+b^2\sin^2\theta)(a^2\cos^2\theta+b^2\sin^2\theta)}d\theta$,

(E) other than above.

II. 計算題部分 四題共 40 分

1. The academy of astronomy declares that the gravitational fields for four unknown planets are as follows. Please justify if the gravitational fields on those planets are conservative. (10%)

α planet: $x^2\mathbf{i}+y^2\mathbf{j}+z\mathbf{k}$

β planet: $y\cos xy\mathbf{i}+x\cos xy\mathbf{j}+\sin z\mathbf{k}$

γ planet: $x^2\mathbf{i}+z^2\mathbf{j}+y^2\mathbf{k}$

δ planet: $yz\mathbf{i}+xz\mathbf{j}+xy\mathbf{k}$

2. If $x(t)$ and $y(t)$ are orthogonal, then show that the energy of the signal $x(t)+y(t)$ is identical to the energy of the signal $x(t)-y(t)$. Note that the energy for a signal $f(t)$ is denoted as E_f and is defined as (7%)

$$E_f = \int f^2(t)dt$$

3. (8%) Use two different approaches to determine the convolution integral

$$f(t)*g(t) = \int_0^t f(\tau)g(t-\tau)d\tau$$

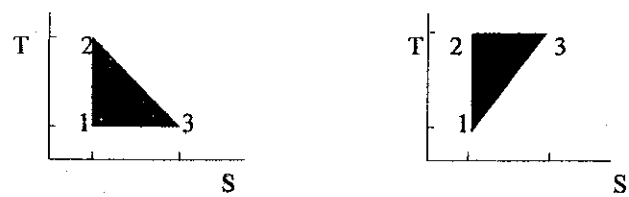
for $f(t) = e^{-2t}$ and $g(t) = 2t$.

4. Solve the initial-value problem (15%)

$\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y} = 2xu$, Initial Condition: $u(0,y) = y-y^2$.

Thermodynamics: (65%)

1. Explain the differences between ideal Rankine cycle and ideal Brayton cycle for power plant. Why do we usually prefer to use Rankine cycle rather than use Brayton cycle in generating electrical power? (5%)
2. Why are the symbols ΔU, ΔKE, ΔPE used to denote the energy change during a process, but the work and heat transfer for the process represented, respectively, simply as W and Q and not as ΔW and ΔQ? (5%)
3. Please explain how to obtain the latent heat of a pure substance from the following measurable properties: temperature, pressure, and specific volume at saturated states (you can use Clapeyron equation to explain it)? (5%)
4. Explain the zeroth law, the first law, and the second law of thermodynamics. What are the contributions of above three laws, respectively? (5%)
5. What are the difference among heat engines, refrigerators, and heat pumps? What are their applications, respectively? Discuss them from their working principles and derive their idea thermal efficiencies by using both high thermal reservoir T_H and low thermal reservoir T_L? (10%)
6. The two power cycles shown to the same scale in the figure are composed of internally reversible processes. Compare the net work developed by these cycles. Which cycle has the greater thermal efficiency? (5%)



7. A certain gas which obeys Nobel-Abel equation of state,

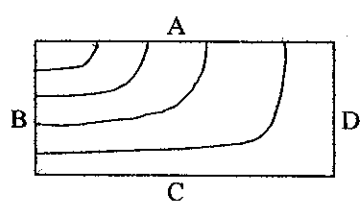
$$P = \frac{RT}{v-b}$$

where b is covolume coefficient. Please find the changes of enthalpy and entropy from state 1 to state 2 in terms of measurable properties c_p, pressure, temperature, and specific volume. (15%)

8. For an ideal vapor-compression refrigeration cycle. (15%)
   (a) Draw a T-s diagram. Describe each process for the cycle.
   (b) How do you calculate heat and work for each process of the cycle?
   (c) Which process provides refrigeration effect?
   (d) What is the COP of the refrigeration cycle?
   (e) If the cycle is used as a heat pump, what is the COP?

Heat Conduction and Radiation: (35%)

9. Write down the heat conduction equation for a 2-D unsteady non-uniform conductivity problem. (5%)
10. Define the fin effectiveness and the fin efficiency. (5%)
11. What is the Lumped-Capacitance-Method (LCM)? When the LCM method is valid? (5%)
12. Define the Biot number. What is the physical meaning of Biot number? (5%)
13. What are the methods for solving a Laplace equation for a 2-D conduction problem? (5%)
14. Which law is used to prescribe the thermal radiation? Write down the law. (5%)
15. Among four control surfaces A, B, C and D of the following figure, which surfaces have heat flux and which surface has highest heat flux? Isothermal lines are shown in the figure. (5%)



(1) For a uniform flow over a flat plate, plot the local heat transfer coefficient along the plate (i.e. h-x) and discuss the shape of the curve that you have drawn. (10%)

(2) Define each of the following numbers, and state their physical interpretation.
(1) Prandtl number, Pr (5%)
(2) Grashof number, Gr (5%)

(3) 下面是有關於圓管內層流(laminar flow)流場的問題（假設流體之性質為常數）

(a) 何謂熱全展流(thermally fully developed flow)？(5%)

(b) 證明熱全展流時之 Nusselt number(Nu)為常數。(10%)

(c) 畫圖示出熱對流係數(h)沿著流向(x)自管入口至熱全展流之變化。(7%)

(d) 畫圖示出等壁面熱通量時，壁溫（T_s）及流體平均溫度(T_m)，沿著流向(x)自管入口至熱全展流之變化情形。(8%)

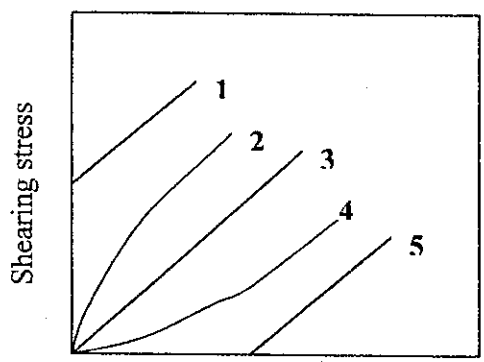
(4) (9%)

(a) What is a fluid? i.e. give definition of a fluid?

The relation of the rate of shearing strain and shearing stress for five materials are described in the following figure.

(b) List the material(s) which can be classified as fluid?

(c) List the material(s) which can be classified as Newtonian fluid?



(5) (12%)

The x and y components of a velocity field are given by $u=xy$, $v=-xy$.

(a) Determine the equation for the streamlines of this flow

(b) Draw the streamlines, in the x-y plane, which pass through the points (0,0), (1,1), and (2,2). Indicate the flow directions on the streamlines

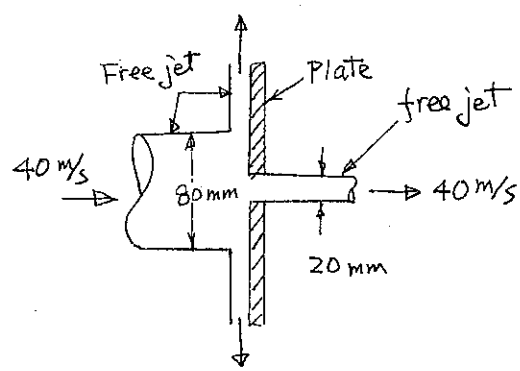
(c) Determine the x and y components of the local acceleration at the point (2,2)

(d) Is the flow irrotational, or rotational? Explain.

(6) Consider mass conservation of a control volume $dx \times dy \times dz$ in a fluid flow, derive the continuity equation (i.e. the differential equation for conservation of mass) (10 %)

(7) Consider the natural convection problem of a vertical plate with a size L×L, immersed in a fluid. The heat transfer coefficient for natural convection, h, is a function of L, g, β, (T_s-T_∞), ν, and α.
Where g=acceleration, β=coefficient of expansion, (T_s-T_∞)=temperature difference, ν=kinematic viscosity, and α=thermal diffusivity (11 %)
(a) What are the primary dimensions in the expressing the physical quantities?
(b) Apply the Buckingham π theorem, express the dimensionless heat transfer coefficient (Nusselt number) in terms of dimensionless groups

(8) A circular plate having a diameter of 300 mm is held perpendicular to an axisymmetric horizontal jet of air having a velocity of 40 m/s and a diameter of 80 mm as shown in the figure below. A hole at the center of the plate results in a discharge jet of air having a velocity of 40 m/s and a diameter of 20 mm. Determine the horizontal component of force required to hold the plate stationary. The air has a density of 1.23 kg/m^3. (8 %)



Prob. #1 (10%)

Please list at least two reasons why can we simply adopt the conventional (engineering) stress-strain diagram (curve) for metals, instead of true stress-strain diagram, in mechanical design and applications.

Prob. #2 (20%)

Data taken from a stress-strain test are given in the Table 1. The curve is linear between the origin and the first point. Plot the diagram, and determine the modulus of elasticity, modulus of resilience, and modulus of toughness.

Table 1

σ (ksi)	ε (in./in.)
0	0
32.0	0.0016
33.5	0.0018
40.0	0.0030
41.2	0.0050

Prob. #3 (20%)

(1) The principle of superposition is often used to determine the stress or displacement at a point in a member when the member is subjected to a complicated loading. Please answer under what conditions the principle of superposition can be applied. (10%)

(2) Determine the absolute maximum shear stress in the shaft as shown in Fig.1 with both ends A and D fixed. (10%)

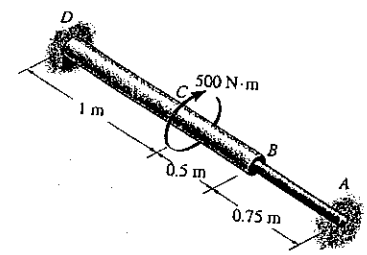


Fig. 1

Prob. #4 (25%)

The principal stresses acting at a point in a body are shown in Fig.2. Draw the three Mohr's circles that describe this state of stress, and find the maximum in-plane shear stresses and associated average normal stresses for the x-y, y-z, and x-z planes. For each case, show the results on the element oriented in the appropriate direction.

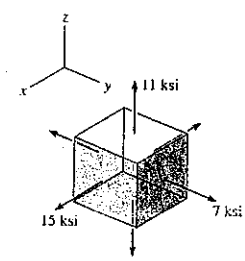


Fig. 2

Prob. #5 (25%)

The wooden beam (Fig. 3) has a rectangular cross section and is used to support a load of 1200 lb. If the allowable bending stress is σ_{allow} = 2 ksi and the allowable shear stress is τ_{allow} = 750 psi, determine the height h of the cross section to the nearest 1/4 in. if it is to be rectangular and have a width of b = 3 in. Assume the supports at A and B only exert vertical reactions on the beam.

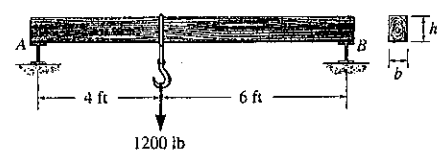


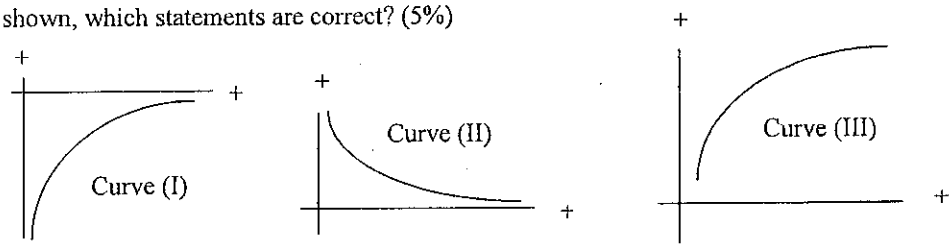
Fig. 3

[i] Multiple Choices Problem

Please chose the correct answers for following 8 problems:

Please be noted that the answers for each problem may be more than one

(1) Consider the motion of a particle on an axis. Let x, v, & a are the displacement, velocity, and acceleration of the particle, respectively, and t is the time. Then for the following 3 figures shown, which statements are correct? (5%)



(A) If curve (I) is a graph with ordinate a and abscissa x, then curve (II) is a graph with ordinate v and abscissa x.

(B) If curve (II) is a graph with ordinate v and abscissa t, then curve (III) is a graph with ordinate x and abscissa t.

(C) If curve (III) is a graph with ordinate a and abscissa t, then curve (I) is a graph with ordinate v and abscissa t.

(D) If curve (III) is a graph with ordinate a and abscissa x, then curve (II) is a graph with ordinate v and abscissa x.

(E) None of the previous statements is correct.

(2) Consider the kinematics of particles, which statements are correct? (5%)

(A) The displacement of a particle and the distance it travels on its path are the same thing.

(B) The acceleration vector of a particle is perpendicular to the path of the particle.

(C) The velocity vector of a particle is tangent to the path of the particle.

(D) The finite angular displacement and the angular velocity are vector quantities.

(E) None of the previous statements is correct.

(3) Consider the kinetics of particles, which statements are correct? (5%)

(A) A particle that subjected to constant external forces is in equilibrium.

(B) Any reference frame that translate relative to a Newtonian reference frame is itself a Newtonian reference frame.

(C) Newton's second law, $\mathbf{F} = \mathbf{ma}$, is only valid in a non-accelerating reference frame.

(D) For a particle P subjected to a force directed toward a fixed point O, the position vector **r** from O to P sweeps out equal areas in equal times.

(E) None of the previous statements is correct.

(4) Consider a mechanical system subjected to several external forces and moves from the configuration 1 to configuration 2. Let T_1 & T_2 be the kinetic energy of the system corresponding to configurations 1 & 2, respectively. Also, let V_1 & V_2 be the potential energy of the system corresponding to configurations 1 & 2, respectively. $U_{1\text{-}2}^{C}$ & $U_{1\text{-}2}^{NC}$ are the work performed on the system by conservative and non-conservative forces, respectively, as the system moves from the configuration 1 to configuration 2. Then, which statements are correct? (5%)

(A) $U_{1\text{-}2}^{C} + U_{1\text{-}2}^{NC} = T_2 - T_1$

(B) $U_{1\text{-}2}^{C} = V_2 - V_1$.

(C) If $U_{1\text{-}2}^{C} = 0$ & $U_{1\text{-}2}^{NC} \neq 0$, then $T_1 + V_1 = T_2 + V_2$

(D) If $U_{1\text{-}2}^{C} \neq 0$ & $U_{1\text{-}2}^{NC} \neq 0$, then $T_1 + V_1 + U_{1\text{-}2}^{NC} = T_2 + V_2$.

(E) None of the previous statements is correct.

(5) Consider the momentum principles, which statements are correct? (5%)

(A) The linear momentum of a particle in a system of particles is not affected by internal forces.

(B) In an isolated system, linear momentum is conserved.

(C) If the moment of momentum of a particle with respect to an arbitrary point Q is conserved, the net force on the particle must be zero.

(D) The sum of the moments, about any point Q, of the external forces that act on a system of particles is equal to the time rate of change of the moment of momentum relative to Q if Q has constant velocity relative to a Newtonian frame.

(E) None of the previous statements is correct.

(6) Consider the differential equation of motion for a vibrating SDOF (Single Degree Of Freedom) system: $A\ddot{x} + B\dot{x} + Cx = F(t)$. Then, which statements are incorrect? (5%)

(A)The constant C is the effective damping constant of the system.

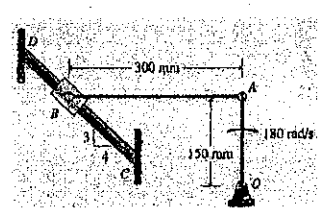
(B)The constant A is the effective mass of the system.

(C) Because there is only one coordinate needed to describe the SDOF system, the system must translate.

(D) The $B\dot{x}$ term in a vibrating system acts to return the system to its static equilibrium position.

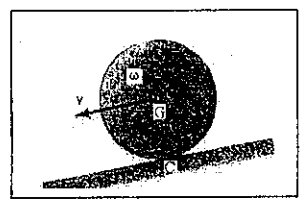
(E) All of the previous statements is correct.

(7) In the mechanism shown in RHS figure, crank OA rotates clockwise with a constant angular speed of 180rad/s. Link AB is pinned at A, and its other end is constrained to move on the inclined guide CD. Then, which statements are correct? (10%)



(A) The distance between the instantaneous center of rotation and point A is 400mm.
(B) The speed at point A is 27000m/s.
(C) The link AB rotates counterclockwise.
(D) The velocity of point B is 33750mm/s directed toward to point D.
(E) None of the previous statements is correct.

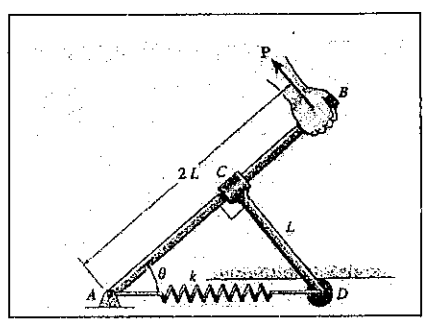
(8) A sphere , a cylinder and a hoop , each having the same mass & the same radius , are released from rest on an incline with rough surface. After they have rolled without slipping through a distance corresponding to a change in elevation h,which statements are correct? (10%)



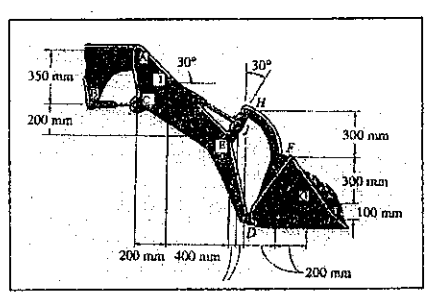
(A) The instantaneous center of rotation is located at the centroid G.
(B) The friction force does no work.
(C) If k is the radius of gyration with respect to a perpendicular axis through the mass center G, then the larger the value of k, the smaller the velocity of the body.
(D) The hoop attains the smallest velocity
(E) None of the previous statements is correct.

[ii] A golf ball, dropped in a vacuum from a height of 3 m onto a horizontal floor, bounces to the height of 2 m. The time that elapses during this displacement is 1.48 s. Calculate the time [s] that the ball remains in contact with the floor ($g = 9.81\ m/s^2$) (10%)

[iii] The two-bar mechanism consists of a lever arm AB and smooth link CD, which has a fixed collar at its end C and a roller at the other end D. Determine the force **P** needed to hold the lever in the position θ. The spring has a stiffness k and unstretched length $2L$. The roller contacts either the top or bottom portion of the horizontal guide. (20%)



[iv] The tractor shovel carries a 500-kg load of soil, having a center of gravity at G. Compute the forces developed in the hydraulic cylinders IJ and BC due to this loading. (20%)



1. (10%)(a).Please draw the constant damping-ratio loci for $\zeta = 0.1, 0.5$, and 0.9, and draw the constant natural frequency loci for $\omega_n = 0.1, 0.5$, and 0.9 on the s-plane.
   (10%)(b).Please draw the desired region for $\zeta = 0.6$, and $-\zeta\,\omega_n \leqq -2$ on the s-plane.
   (10%)(c). What is the meaning of the factor, $e^{-\zeta\omega_n t}$, in the solution of the standard second-order system of $C(s) = [\omega_n^2/(s^2 + 2\zeta\,\omega_n + \omega_n^2)]R(s)$, where R(s) is the unit-step input, and C(s) is the output..

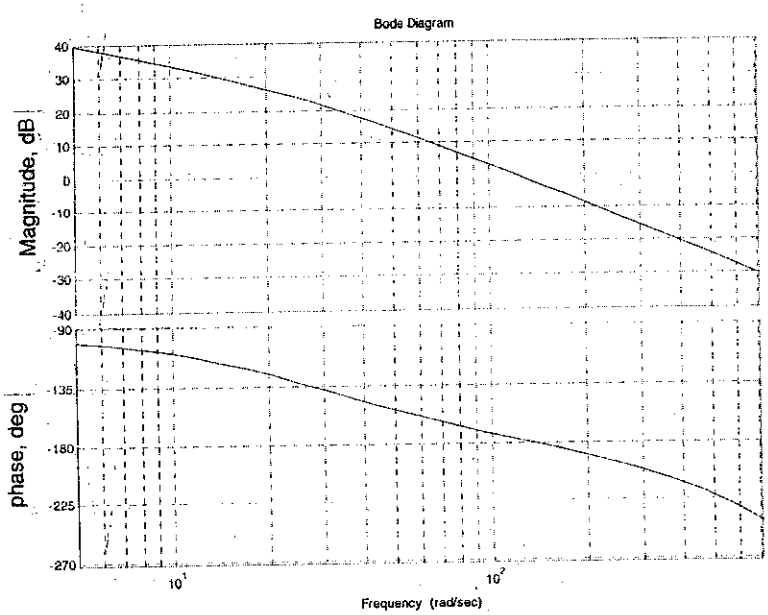
2. An MATLAB M-file program to achieve the open-loop response and negative unity-feedback closed-loop response for a servomotor control system is shown as follows:

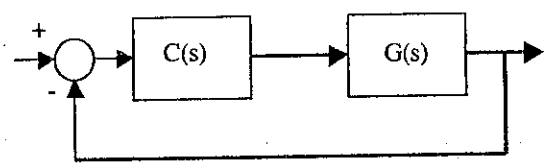
```
%Plotting open-loop response and closed-loop response
Ko=24/pi;    %Ko is a ratio for transfer an angle to a voltage
G1_num=[6e-4]; G1_den=[1.08e-5  8.33e-5  0]; %6e-4 means 6 · (10)^-4
G1=tf(G1_num, G1_den); %tf() means transfer function
n=1/10;      %n is the gear ratio for motor to load
Gs=Ko*G1*n
Hs=tf(1,1);
%plot the open-loop response
sys_open=Gs;
step(sys_open)
gtext('open-loop-output')
hold
%plot the closed-loop response
sys_closed=feedback(Gs,Hs);
step(sys_closed)
gtext('closed-loop-output')
grid
title('unit-step response of a 2nd-order servomotor system')
xlabel('Time,sec')
ylabel('theta,rad.')
hold
```

(10%)(a). Please write down the closed-loop transfer function.
(10%)(b). Please compute the 2% settling time.

3.(15%)Discuss the advantages and disadvantages of controller design based on the frequency response of the system, which is to be controlled.

4.Consider a dynamic system G(s), whose Bode diagram is shown below. This system will be controlled by a unit feedback scheme as indicated. Please answer the following questions.
(10%)(a).Write down the possible transfer function of the dynamic system.
(15%)(b).Design or choose a proper controller C(s) so that the bandwidth of the closed-loop system can be higher than 20 Hz.
(10%)(c).In your viewpoint, can you further increase the bandwidth up to 100 Hz? Why?

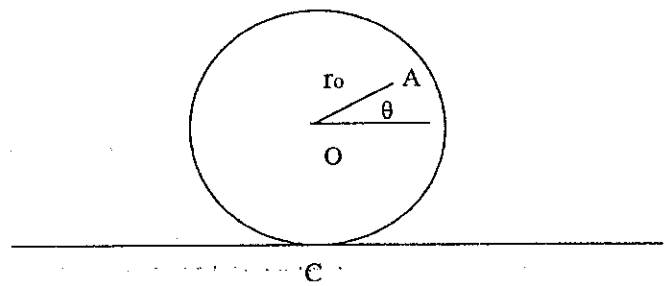




國立中山大學九十二學年度碩士班招生考試試題

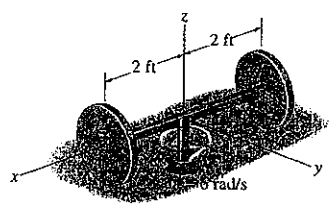
科目：應用力學 (機電系丙組)

1. The wheel of radius r rolls to the left without slipping and at the instant, the center O has a velocity V_0 and an acceleration a_0 to the left. Determine the acceleration of points A and C on the wheel for the instant considered.(25%)

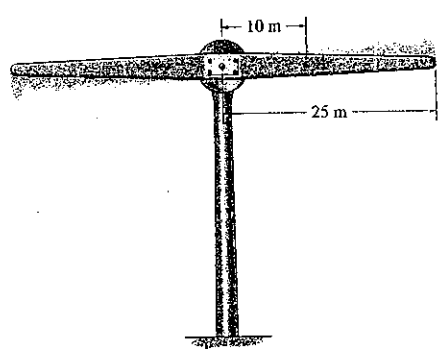


2. A metal hoop with a radius r = 150mm is released from rest on the 20 degrees incline. If the coefficients of static and kinetic friction are 0.15 and 0.12, determine the angular acceleration of the hoop. (25%)

3. Each of the two disks has a weight of 10 lb. The axle AB weighs 3 lb. If the assembly is rotating about the z axis at ω_z=6 rad/s, determine its angular momentum about the z axis and its kinetic energy. The disks roll without slipping. (25%)



4. The lightweight turbine consists of a rotor which is powered from a torque applied at its center. At the instant the rotor is horizontal it has an angular velocity of 15 rad/s and angular acceleration of 8 rad/s^2. Determine the internal normal force, shear force, and moment at a section through A. Assume the rotor is a 50-m-long slender rod, having a mass of 3kg/m. (25%)



一 選擇題（共五十分，每題五分， 答錯每題倒扣二分）

(1) A harmonic motion has an amplitude of 0.05 *m* and a frequency of 10 Hz. The period is: (a) 0.05 *sec;* (b) 0.1 *sec;* (c) 0.2 *sec;* (d) 0.5 *sec;* (e) 1 *sec.*

(2) In Problem (1), the maximum acceleration is:
(a) 39.48 m/s^2, (b) 78.96 m/s^2, (c) 157.91 m/s^2 (d) 197.39 m/s^2, (e) 394.79 m/s^2.

(3) The maximum amplitude and the maximum acceleration of the foundation of a centrifugal pump were found to be $x_{max} = 0.25\,mm$ and $\ddot{x}_{max} = 0.4\,g$ for its harmonic oscillation. The operating speed of the pump is about: (a) 5 *rpm;* (b) 10 *rpm;* (c) 15 *rpm;* (d) 20 *rpm;* (e) 40 *rpm.*

(4) An industrial press is mounted on a rubber pad to isolate it from its foundation. If the rubber pad is compressed 5 *mm* by the self-weight of the press, the natural frequency of the system is about:
(a) 3 Hz; (b) 4 Hz; (c) 5 Hz; (d) 6Hz; (e) 7 Hz.

(5) The natural frequency of a spring-mass system is found to be 2 Hz. When an additional mass of 1 *kg* is added to the original mass *m*, the natural frequency is reduced to 1 Hz. The mass *m* is:
(a) 0.33 *kg;* (b) 0.66 *kg;* (c) 1.32 *kg;* (d) 1.65 *kg;* (e) 1.98 *kg.*

(6) The spring constant *k* in Problem (5) is:
(a) 12.57 *N/m;* (b) 22.68 *N/m;* (c) 52.64 *N/m;* (d) 75.42 *N/m;* (e) 100.56 *N/m.*

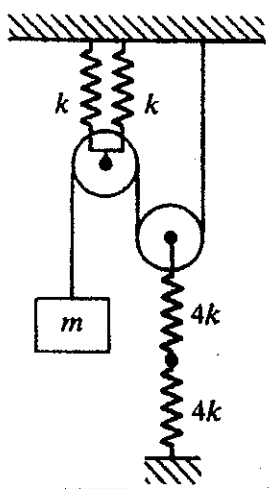


Fig. 1

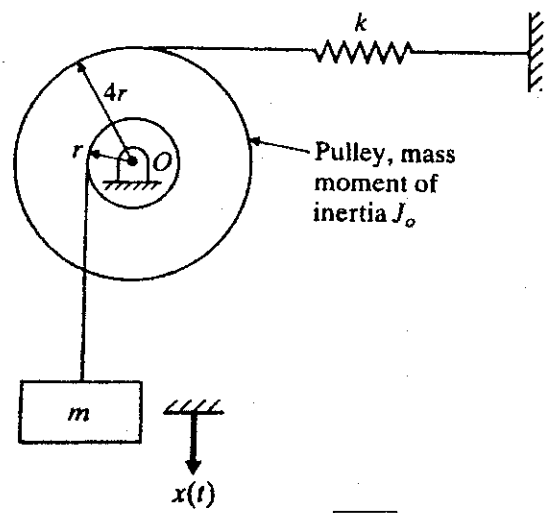


Fig. 2

(7) The equivalent spring constant of the pulley system shown in Fig. 1 by neglecting the friction and the masses of the pulleys is: (a) *k*/4; (b) *k*/2; (c) *k*; (d) 2*k*; (e) 4*k*.

(8) In Fig. 1, the natural frequency of the pulley system is:
(a) $\sqrt{k/4m}$; (b) $\sqrt{k/2m}$ (c) $\sqrt{k/m}$; (d) $\sqrt{2k/m}$; (e) $\sqrt{4k/m}$.

(9) The kinetic energy of the system shown in Fig. 2, is:
(a) $\frac{1}{2}(m+\frac{J_0}{16r^2})\dot{x}^2$; (b) $\frac{1}{2}(m+\frac{J_0}{4r^2})\dot{x}^2$; (c) $\frac{1}{2}(m+16J_0r^2)\dot{x}^2$; (d) $\frac{1}{2}(m+\frac{J_0}{r^2})\dot{x}^2$; (e) $\frac{1}{2}(m+J_0)\dot{x}^2$;

(10) The natural frequency of the system shown in Fig. 2, is:
(a) $\sqrt{\frac{kr^2}{16mr^2+J_0}}$; (b) $\sqrt{\frac{kr^2}{mr^2+4J_0}}$; (c) $\sqrt{\frac{kr^2}{mr^2+J_0}}$; (d) $\sqrt{\frac{kr^2}{mr^2+16J_0}}$; (e) $\sqrt{\frac{16kr^2}{mr^2+J_0}}$

二 簡答計算題（共五十分）

(1) Draw a top (陀螺) spinning with certain precession on the ground. Explain briefly and label the spin, precession, and nutation velocities of the top. (15%)

(2) Define a simple multiple-degree-of-freedom problem in dynamics and use Lagrange's equation to obtain the solution. (20%)

(3) Explain the use of instant centers in the study of planar and spatial kinematics. (15%)

1. For the mechanism and data given, $T_{13} = 50$in-lb, $W_2 = 16.1$ lb, $W_3 = 32.2$lb, determine:
   (a) the cam torque, T_{12}, and
   (b) the forces on the frame at points A and C (F_{21} and F_{31}).
   Assume that there is friction between the cam and follower only, $\mu = 0.13$.
   (25%)

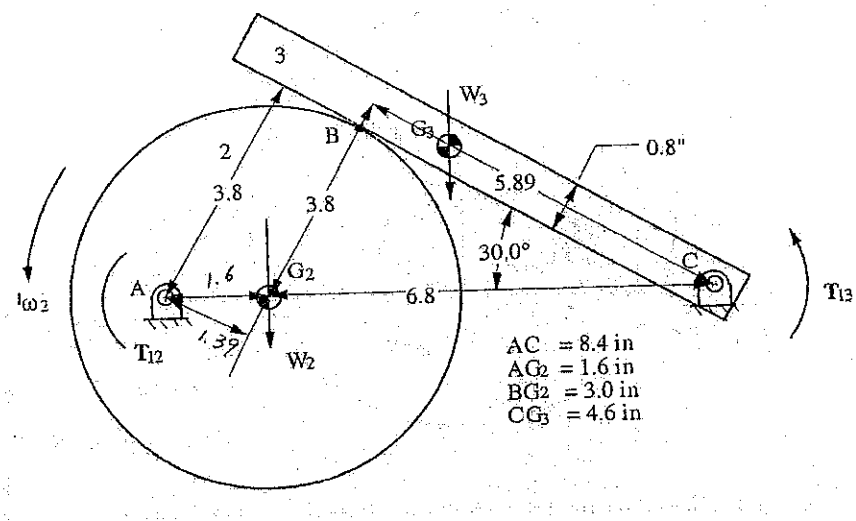


Fig. P1

2. (a) What is the principle of virtual work? (10%)
   (b) Under what conditions you can apply the principle of virtual work? (15%)

3. (a) What are scalars and vectors? (3%)
   (b) What are stable, unstable and neutral equilibrium of force systems? (8%)

4. A system consists of a force **F** acting at the origin O and a couple **M**, where

$$\mathbf{F} = 3\mathbf{i}+6\mathbf{j}+2\mathbf{k}\ (\text{N}) \text{ and } \mathbf{M} = 14\mathbf{i}+7\mathbf{j}+7\mathbf{k}\ (\text{N}\cdot\text{m})$$

   If you represent it by a wrench consisting of the force **F** and a parallel couple $\mathbf{M}_p$, (a) determine $\mathbf{M}_p$, and (b) determine where the line of action **F** intersects x-z plane. (15%)

5. Determine the reaction at the support O and the tension T in the wire AB. (Fig. P4) (12%)

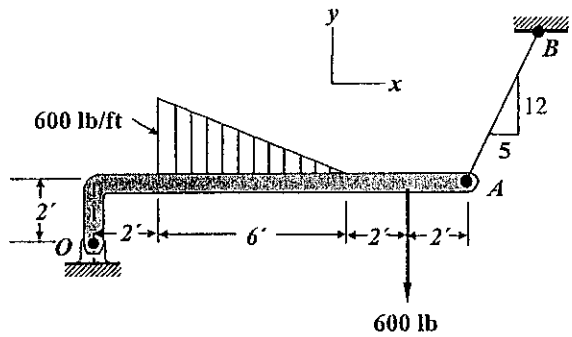


Fig. P4

6. Three blocks of equal weight W are connected, as shown in Fig. P5. Two of them are resting on horizontal planes of the same height. For the position of impending slipping, determine the coefficient of friction μ between the block and the horizontal planes (12%)

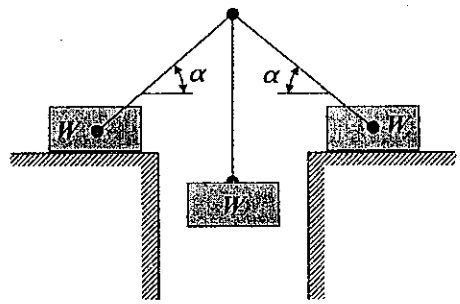


Fig. P5