國立中山大學九十學年度碩博士班招生考試試題

科目：工程數學【機電系碩士班】

1. (15%) Find the eigenvalues and eigenvectors of the matrix and fill out your answers in the blanks.

$$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

(答案不限次序) __A__, __B__, __C__, __D__, __E__, and $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

2. (10%) In the following vectors, which one(s) is(are) linearly independent.

(A) 〔1, 5, 3〕, 〔2, 4, 6〕, 〔3, 9, 11〕
(B) 〔1, 0〕, 〔1, 2〕, 〔3, 4〕
(C) 〔1, 2, 3〕, 〔4, 5, 6〕, 〔7, 8, 9〕
(D) 〔9, 0, 9〕, 〔0, 6, 6〕, 〔3, 3, 0〕
(E) 〔3, 0, 2, 4, 5〕, 〔7, 2, 6, 1, 0〕, 〔1, 2, 2, -7, -10〕

Ans: Linear independent:

3. (15%) Evaluate the following integrals

1. $\int_{-\infty}^{\infty} \frac{\sin x}{1+x^4} dx$ 2. $\int_0^{2\pi} \frac{d\theta}{13-5\sin\theta}$

3. $\oint_C \frac{1}{Z^2+1} dz$ C: $|z+i|=1$, (counterclockwise)

4. (10%) Please give the definition of the following terms

1. analytic function 2. entire function 3. meromorphic function
4. residue 5. Laurent series

5. (12%) Find the eigenvalues and eigenfunctions of the following problem.

$$y'' + \lambda y = 0, y(0) = y(\pi), y'(0) = y'(\pi).$$

6. (13%) Solve the differential equation.

$$y'' + 2y' + y = \delta(t-1), y(0) = 2, y'(0) = 3.$$

7. (25%) Given a curve C: $\mathbf{r}(t) = 3\cos t\,\mathbf{i} + 3\sin t\,\mathbf{j} + 4t\,\mathbf{k}$, find

(a) a tangent vector $\mathbf{r}'(t)$ and the corresponding unit tangent vector $\mathbf{u}(t)$,

(b) $\mathbf{r}'$ and $\mathbf{u}$ at the point $P(0, 3, 2\pi)$,

(c) the tangent at P,

(d) sketch the curve and the tangent,

(e) the length of the curve from (3, 0, 0) to $(-3, 0, 4\pi)$.

1. State (verbally and/or mathematically) as many different forms as you know the first law of thermodynamics. What is the first law used for? (20%)

2. Write down two statements of the second law of thermodynamics and prove that they are equivalent to each other. What is the value of the second law? (20%)

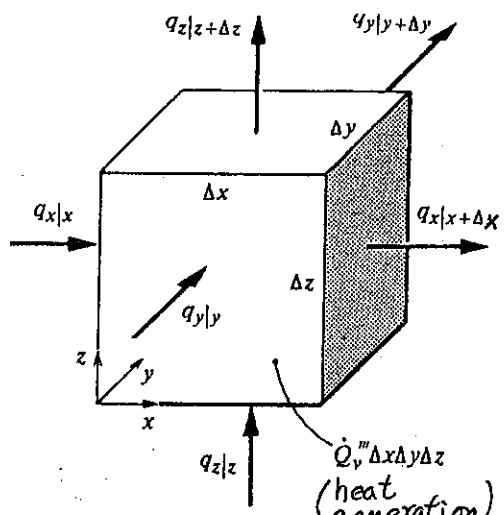
3. Draw P-v and T-s diagrams for each of the following ideal cycles:
   (a) Carnot cycle (4%)
   (b) Stirling cycle (4%)
   (c) Brayton cycle (4%)
   (d) Rankine cycle (4%)
   (e) Otto cycle (4%)

4. Draw a psychrometric chart. (5%)

5. A circular ice rink is 20 m in diameter and is to be temporarily enclosed in a hemispherical dome of the same diameter. The ice is maintained at 270 K, and on a particular day the inner surface of the dome is measured to be 290 K. Estimate the radiant heat transfer from the dome to the rink if both surfaces can be taken as black. (10%)

6. A 0.5 W, 1.5 MΩ graphite resistor has a diameter of 1 mm and is 20 mm long; it has a thin glass sheath and is encapsulated in micanite (crushed mica bonded by a phenolic resin). The micanite serves both as additional electrical insulation and to increase the heat loss. It can be assumed that 50% of the I^2R heating is dissipated by combined convection and radiation from the outer surface of the micanite to surroundings at 300 K with h_o = 16 W/m² K; the remainder is conducted through copper leads to a circuit board. If the conductivity of micanite is 0.1 W/m K, what radius will give the maximum cooling effect, and what is the corresponding resistor temperature? (10%)

7. 請依右圖符号，推導 Unsteady Heat Conduction Equation. (15%)



1. What are the differences of the descriptions between Lagrangian method and Eulerian method? Which method do we usually use to describe the fluid motion? Why? 5%

2. What are the differences in drag force between flow past a streamline body and flow past a blunt body? 5%

3. Why can we divide an external flow field into viscous region and inviscid region? 5%

4. The pressure drop, Δp, for steady, incompressible turbulent flow with viscosity μ and density ρ in a horizontal round pipe of diameter D can be written in functional form as

$$\Delta P = F(V, D, l, \varepsilon, \mu, \rho)$$

where V is the average velocity, l is the pipe length, and ε is a measure of the roughness of the pipe wall, Please find the dimensional groups of this flow. 15%

5. For a laminar flow past a flat surface, the velocity distribution in the x-direction u through the boundary layer can be approximated by a third order polynomial equation,

$$u/U = a_0 + a_1(y/\delta) + a_2 (y/\delta)^2 + a_3 (y/\delta)^3$$

Please write down the boundary conditions of this case to determine the coefficiencies : a_0, a_1, a_2, and a_3. Then find the integral boundary layer solution for this flow and use the result to evaluate the local shear stress and friction drag coefficient.

Note : The Integral equation for laminar flow can be written as

$$\frac{d}{dx}\int_0^\delta u^2 dy - U(x)\frac{d}{dx}\int_0^\delta u dx = -\frac{\delta}{\rho}\frac{dP}{dx} - \nu\left(\frac{\partial u}{\partial y}\right)_0$$

where U(x) is freestream velocity, y is the coordinate in the y direction, ρ is the density of fluid, δ is the boundary layer thickness, ν is the kinematic viscosity of the fluid. 20%.

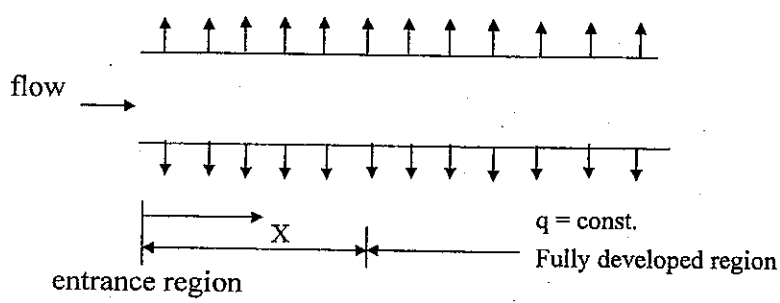
6 (8%) Consider hydrodynamically and thermally fully developed flow in a circular tube of constant cross-section. If the flow is laminar and the fluid properties are assumed to be temperature independent, what happens to the heat transfer coefficient if (a) the flow rate is doubled and the flow is still laminar, (b) the thermal conductivity

is doubled, (c) the viscosity is doubled, (d) the diameter of the circular duct is doubled, while the flow rate is the same?

7 (8%) For laminar flow in a circular tube, the distance from the inlet at which the temperature profile approaches its fully developed shape is called thermal entrance length. What happens to the thermal entrance length if (a) the flow rate is doubled and the flow is still laminar, (b) the thermal conductivity is doubled, (c) the viscosity is doubled, (d) the diameter of the circular duct is doubled, while the flow rate is the same?

8 (8%) Sketch the distribution of bulk temperature T_b and surface temperature T_s of a tube as a function of X

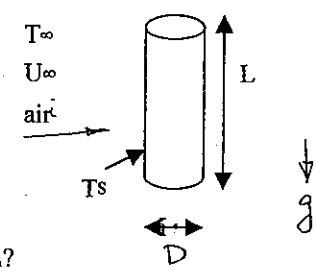
(a) in case the heat flux out of the tube is constant.



(b) in case the surface temperature is constant and the entrance temperature of the flow is higher than the surface temperature.

9 (10%) For a vertical cylinder in a horizontal air flow, with kinematics viscosity ν, thermal diffusivity α, volumetric coefficient of thermal expansion β. Gravitational acceleration is g.

(a) List the important dimensionless parameter(s) for the forced convection problem
(b) List the important dimensionless parameter(s) for the natural convection problem
(c) Under what condition natural convection is more important than forced convection in this problem? (use the symbols appear in the question)



10 (16%) For a long structural component of a bridge has the cross section shown in the following figure.

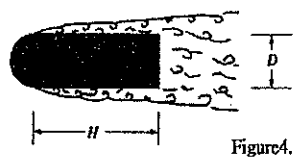


Figure4.

D= 0.2m, H= 0.6m, and a representative wind velocity is 10 m/sec. Standard air can be assumed. The shedding frequency is to be determined through the use of a small-scale model that is to be tested in a water tunnel. We expect shedding frequency $\omega = f(D, H, V, \rho, \mu)$, (a) what dimensionless parameters would you use to organize these data? (b) for the model H_m =120mm and the water temperature is 20℃, $\mu = 1 \times 10^{-3} kg / m \cdot s$, $\rho = 998 kg / m^3$. Determine the model dimension Dm, and the velocity at which the test should be performed. (c) If the shedding frequency for the model is found to be 49.9Hz, what is the corresponding frequency for the prototype? (d) If the drag per unit length on the model is measured to be 1Nt, predict the drag per unit length on the prototype.

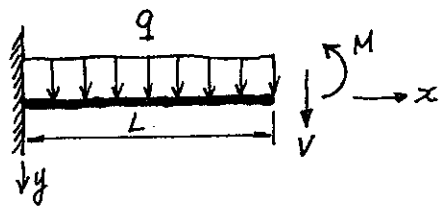
(For standard air $\mu = 1.79 \times 10^{-5} kg / m \cdot s$, $\rho = 1.23 kg / m^3$)

國立中山大學九十學年度碩博士班招生考試試題

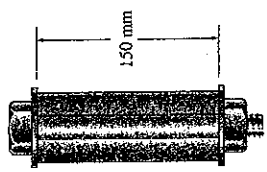
科目：材料力學【機電系碩士班】乙組

1. (25%) 我們經常用 $\sigma=My/I$ 來求 Beam 的 stress，請問
   a. 此公式各個量所代表的意義。畫圖說明，並標出座標系統。
   b. 此公式是在何種假設下導出的？
   c. 證明 neutral axis 通過 Beam 截面的形心。

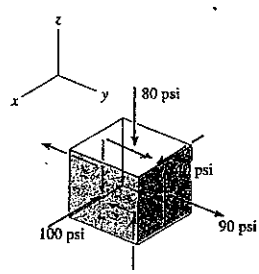
2. (25%) Cantilever Beam 如圖所示，q 為 distributed force，M 及 V 為作用於端點之 bending moment 及 shear force.
   a. 請列出所有的邊界條件
   b. 試求 displacement $w=w(x)$
   （需要的物理量請自行假設）



3. (25%) 下圖所示的結構中，套筒之材質為編號 2014-T6 之鋁管(E_{al}=73.1 GPa，$\alpha_{al}=23\times10^{-6}/°C$)，其截面積為 600 mm^2，中間之螺栓材質則為編號 A-36 之不鏽鋼(E_{st}=200 GPa，$\alpha_{st}=12\times10^{-6}/°C$)，其截面積為 400 mm^2。當溫度為 T_1=15°C 時，螺帽鎖固的位置正好使得整體組合並未受到軸向力的作用，請問當溫度為 T_2=80°C 時，螺栓及套筒中的平均正向應力為何?



4. (25%) 考慮下圖所示的應力狀態，請問其相對應的 Mohr's circles、主應力、以及絕對最大剪切應力為何？



Part I: Please chose the correct answers for following problems: (Please be noted that the answer for each problem may be more than one) **(55%)**

(1) Let $\Sigma \boldsymbol{F}$ be the resultant force vector acting on the particle P, and m , $\boldsymbol{a}$ are the mass and acceleration vector of particle P, respectively. Which statements are correct? (5%)

(A) If $\Sigma \boldsymbol{F} = 0$, then $\boldsymbol{a} = 0$.

(B) $\Sigma \boldsymbol{F}$ is in parallel with $\boldsymbol{a}$.

(C) $\Sigma \boldsymbol{F} = m\boldsymbol{a}$, where $\Sigma \boldsymbol{F}$ is in Newton, m is in kilogram, and $\boldsymbol{a}$ is in km/hr^2.

(D) If $\boldsymbol{a}$ represents a relative acceleration measured with respect to a rotating coordinate system, the equation, $\Sigma \boldsymbol{F} = m\boldsymbol{a}$, still holds.

(E) None of the previous statements is correct.

(2) Consider the central-force motion of an object about the sun. Let (r, θ) be the polar coordinates (with the origin fixed at the center of the sun) of the object, M be the mass of the sun, and G is the universal constant of gravitation. Then the path of motion of the object can be described as

$$(1/r) = (G)(M)[1+(e)(\cos\theta)]/[(r^2)(d\theta/dt)]^2$$

where e is called the eccentricity. Which of the following statements is correct? (5%)

(A) If $e > 1$, then the path is a parabola.

(B) If $e = 1$, then the path is a hyperbola.

(C) If $e < 1$, then the path is an ellipse.

(D) If $e = 0$, then the path is a circle.

(E) None of the previous statements is correct.

(3) Consider the motion of a system of particles, $P_1, P_2, \ldots P_i, \ldots P_j, \ldots P_n$. Let m_i and $\boldsymbol{a}_i$ be the mass and acceleration of particle P_i, respectively, and the vectors $m_i\boldsymbol{a}_i$ $(1 \le i \le n)$ are referred as the effective forces on the particles. Also let $\boldsymbol{f}_{ij}$ be the internal force vector exerted on P_i by P_j, and $\boldsymbol{F}_i$ is the resultant of all the external forces on P_i. Which statements are correct? (5%)

(A) The system of the external forces acting on the particles and the system of the effective forces, $m_i\boldsymbol{a}_i$, of the particles are equipollent.

(B) The system of the internal forces $\boldsymbol{f}_{ij}$ is equipollent to zero.

(C) The internal forces $\boldsymbol{f}_{ij}$ have no effect on the particles under consideration.

(D) Two systems of external forces that have the same force resultant and the same moment resultant will have the same effect on a given system of particles.

(E) None of the previous statements is correct.

(4) Consider a rigid slab is in plane motion. Let $\mathbf{a}_G$ be the acceleration vector of mass center G of the slab, and α is the angular acceleration vector of the slab. Which statements are correct? (5%)

(A) If the slab is in translation, then $\mathbf{a}_G = 0$.

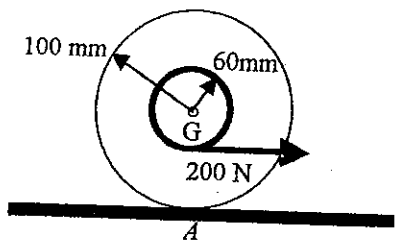
(B) If the slab rotates about a fixed axis perpendicular to the slab and passing through point G, then $\alpha = 0$.

(C) (The moment resultant about point G) = $(I_G)(\alpha)$, where I_G is defined as the moment of inertia about centroidal axes perpendicular to the slab.

(D) If the slab is constrained to rotate about a fixed axis which pass through an arbitrary point A and perpendicular to the slab, then (The moment resultant about point A) = $(I_A)(\alpha)$, where I_A is defined as the moment of inertia about the axis of rotation.

(E) None of the previous statements is correct.

(5) A cord is wrapped around the inner drum of a wheel and pulled horizontally with a force of 200 N. The wheel has a mass of 50 kg and a radius of gyration of 70 mm. Knowing that $\mu_s = 0.2$ and $\mu_k = 0.15$, and let F and N are friction force and normal force at contact point A, respectively. Which statements are correct? (10%)



(A) The moment of inertia about the centroidal axis of the wheel equals to 0.245 kg · m^2.

(B) $F \geqq 140$ N and $N \geqq 485$ N

(C) 1.7 m/s^2 ≦ acceleration of the centroid G ≦ 2.1 m/s^2

(D) 17 rad/s^2 ≦ angular acceleration of the wheel ≦ 21 rad/s^2

(E) None of the previous statements is correct

(6) A 600-lb crate is supported by a rope-and-pulley arrangement as shown. The tension in the rope is (5%)

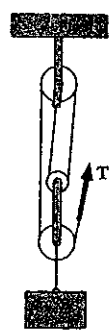
(A) 600-lb

(B) 300-lb

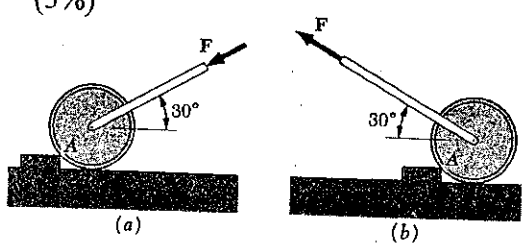
(C) 200-lb

(D) 150-lb

(E) None of the previous statements is correct

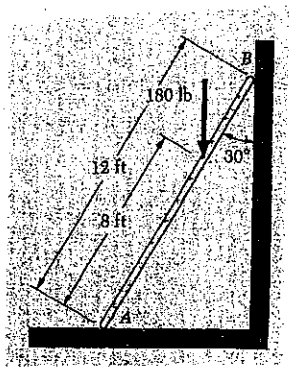


(7) A 600-lb roller, 2 ft in diameter, is used on a lawn. If we try to make it roll over a 3-in obstruction by pushing the roller as shown in Fig (a) or by pulling the roller as shown in Fig. (b). (5%)



(A) The required force F in Fig. (a) is larger than force F in Fig. (b)
(B) The required force F in Fig. (b) is larger than force F in Fig. (a)
(C) The required forces F in Fig. (a), Fig. (b) are the same
(D) None of the previous statements is correct

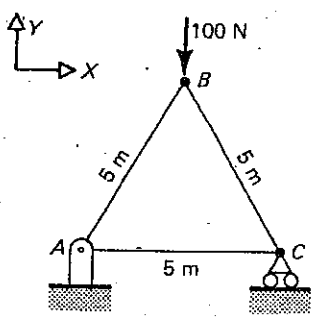
(8) A 12-ft ladder weighting 40 lb is placed against a vertical wall as shown. Knowing that the coefficients of friction between the ladder and the wall is $\mu_w = 0.20$. As a 180-lb man reaches a point 8ft from the lower end A, the ladder is just about to slip. The coefficient of friction μ_f between the ladder and the floor is (5%)



(A) $\mu_f = 0.15$ (B) $\mu_f = 0.20$ (C) $\mu_f = 0.25$ (D) $\mu_f = 0.35$

(E) None of the previous statements is correct

(9) A plane truss (as shown in Figure) is loaded 100 N at point B. The magnitudes of the internal force in member AC is (5%)



(A) $F_{AC} = 50\,\text{N}$ in tension

(B) $F_{AC} = 57.74\,\text{N}$ in compression

(C) $F_{AC} = 28.87\,\text{N}$ in tension

(D) $F_{AC} = 57.74\,\text{N}$ in tension

(E) None of the previous statements is correct

(10) To point out which one in the following statements is incorrect (5%)
(A) Virtual displacements are fictitious displacements
(B) Virtual displacements are very small in magnitude and don't Violate the constraints of a system
(C) A body is in equilibrium if the virtual work done by all the applied forces and moments is zero.
(D) Static equilibrium may be stable or unstable, depending upon the change in the potential energy of a system about the equilibrium position:

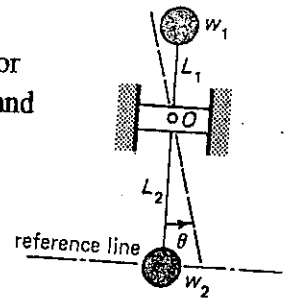
Static equilibrium: $\frac{dV}{dx} = 0$ *and* $\frac{d^2V}{dx^2} < 0$

Unstable equilibrium: $\frac{dV}{dx} = 0$ *and* $\frac{d^2V}{dx^2} > 0$

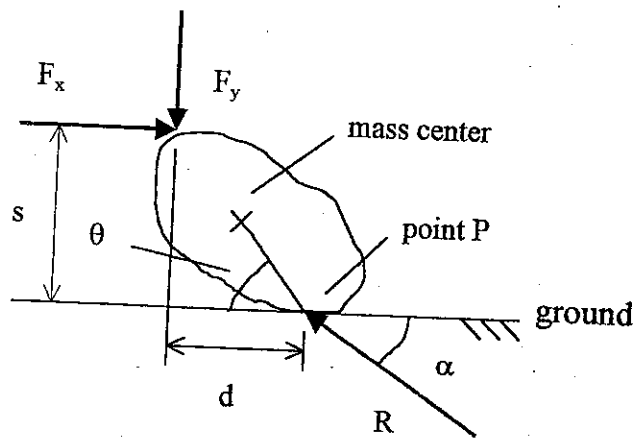
(E) None of the previous statements is incorrect

Part II: (45%)

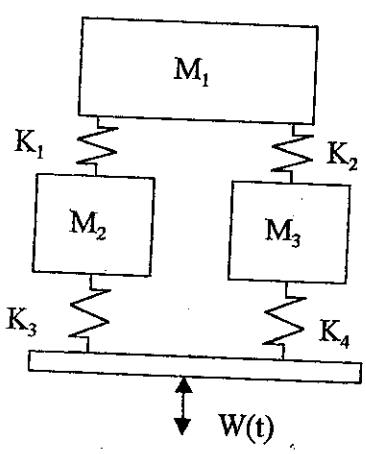
11. Find the condition for equilibrium as well as the type of equilibrium for the system as shown. Assume that the system lies in a vertical plane and is pinned at the joint *O*. (15%)



12. Consider a rigid body with an irregular shape. This body is pressed against the ground at point P by the external forces F_x and F_y. The reaction from the ground has a magnitude R and points to the body with an angle α as indicated. Find the equation of motion of this rigid body at this configuration. Please define the necessary variable(s) or physical term(s) that may help your derivation. (15%)



13. Consider a mechanical vibration system as shown below. Three masses are connected to each other by four springs with spring constant K_1, K_2, K_3 and K_4, respectively. Two of the springs are further connected to a plate. The plate has a specific motion denoted as $w(t)$. Please derive the governing equation of motion for the system. (15%)



國立中山大學九十學年度碩博士班招生考試試題

科目：自動控制【機電系碩士班】丙組

1. (10%) Describe the physical meaning of phase margin. You should consider the interaction between time-delay and system stability.

2. (10%) After increasing the bandwidth of his closed-loop system design, John discovers that he has successfully increased the speed of response of the system. Explain the reason behind this success.

3. (10%) Encouraged by this success, John believes that he can make the speed of response of the closed-loop system even faster by further increasing the bandwidth of the closed-loop system. If this logic is correct, John can increase the speed of the system without limitation. Apparently, this is physically impossible. Why?

4. (10%) In developing a control law for a motor system, it is found that one can use PI or PD controller to achieve the desired control goal. Based on the consideration of measurement accuracy, Mary tells you that she prefers PI controller. Do you know why?

5. (10%) You are told not to perform pole-zero cancellation for an unstable system. What is the reason behind this advice?

6. The dynamics of a well-designed DC servo motor is described by the following equations:

$$\text{motor torque}: \quad \tau = 0.05i = 0.005\dot{\omega} + B\omega$$
$$\text{motor voltage}: \quad v = 2i + 0.05\omega$$

where i is the electric current, ω denotes the rotational speed of the rotor, and B is the damping coefficient, which is usually very small and should be neglected.

You only have two controllers in hands and both are with unit gains. One is a voltage controller, $v = \omega_d - \omega$; and the other is a current controller, $i = \omega_d - \omega$, where ω_d is the input command for the motor's speed.

(1) (20%) Sketch magnitude parts of the **closed-loop** frequency responses using the above two different controllers.

(2) (20%) Determine corresponding bandwidths and steady state errors for a unit step input.

(3) (10%) Which controller would you select to achieve better control performance? Why?

國立中山大學九十學年度碩博士班招生考試試題

科目：動力學【機電系碩士班】丁組

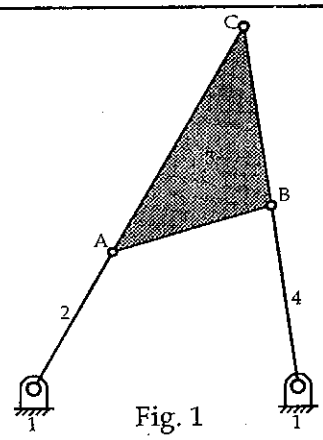


Fig. 1

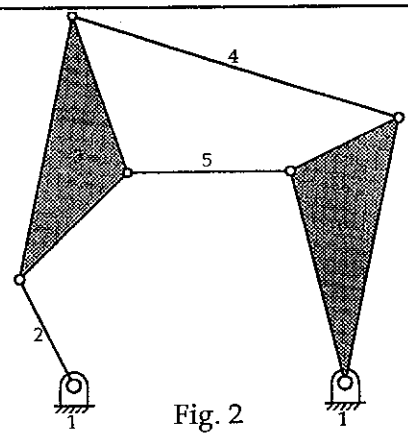


Fig. 2

(1) Both input links (i.e., Link 2) in the above figures rotate clockwise with a constant angular velocity $\omega_2 = 100$ rad/sec. Please compute the angular velocities of these output links: (a) Link 3 in Fig. 1 (10%), (b) Link 6 in Fig. 2. (15%)

(2) Propose a solution for each of the following problems: (a) How to locate a point having a specified acceleration A in a moving rigid-body (e.g., Link 3 in Fig. 1)? (10%), and (b) How to balance the shaking forces and moments due to d'Alembert's accerlations of a rotating 4-bar linkage? (15%)

(3) Referring to Fig. 3, let a mass m having a translational velocity $\dot{x}$ be coupled to another mass (of mass moment of inertia J_0) having a rotational velocity $\dot{\theta}$, as in the rack and pinion arrangement shown below. These two masses can be combined to obtain either a single equivalent translational mass m_{eq} or a single equivalent rotational mass J_{eq}. (a) Determine m_{eq}. (10%) (b) Determine J_{eq}. (10%)

(4) Show the types of damping, (a), (b), (c), and (d) indicated in Fig. 4 for the free vibrations of a single degree of freedom system. (10%)

(5) In the spring-mass system shown in Fig. 5, (a) determine the natural frequency without considering the mass of the spring (10%); (b) determine the natural frequency with considering the mass of the spring m_s (10%).

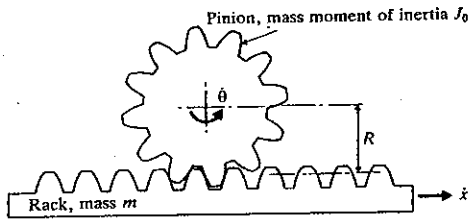


Fig. 3

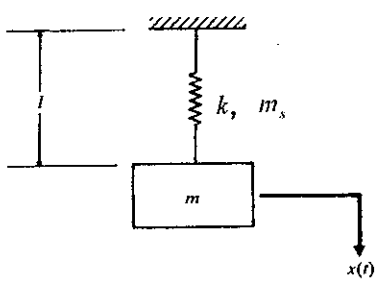


Fig. 5

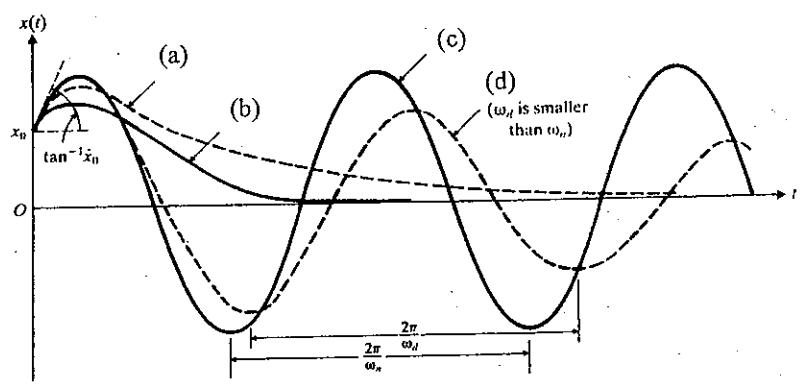


Fig. 4

國立中山大學九十學年度碩博士班招生考試試題

科目：靜力學【機電系碩士班】丁組

1. Find the forces in members CD, DF, and EF of the transmission line truss shown in Fig. 1. (16%)

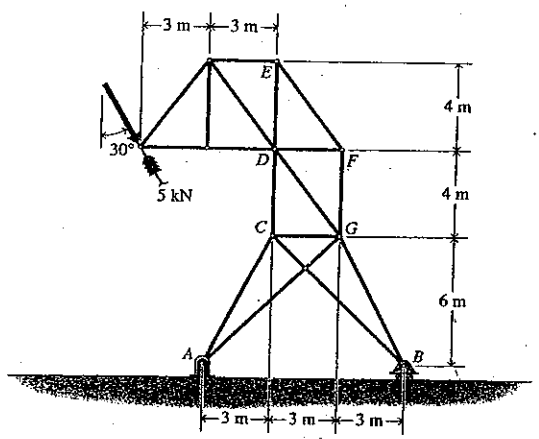


Fig. 1

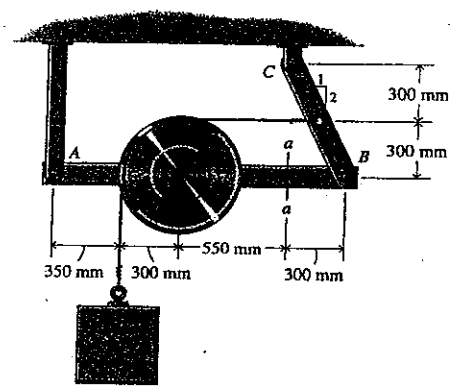


Fig. 2

2. Two bars, a pulley, and a cable are used to support a block as shown in Fig. 2. The two bars have negligible weight. The mass of the pulley is 50 kg and the mass of the block is 100 kg. Determine the internal resisting forces and moment transmitted by cross section *aa* in bar *AB*. (16%)

3. The brake shown in Fig. 3 is used to control the motion of block *B*. If the mass of block *B* is 25 kg, and the kinetic coefficient of friction between the brake drum and brake pad is 0.30, determine the force **P** required for a constant velocity descent. (18%)

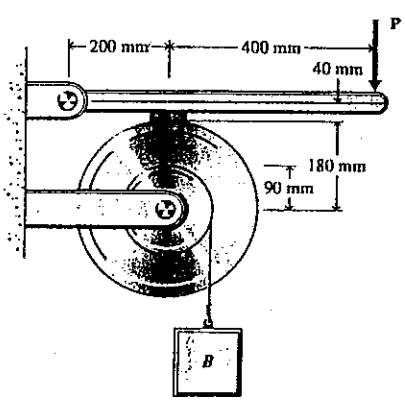


Fig. 3

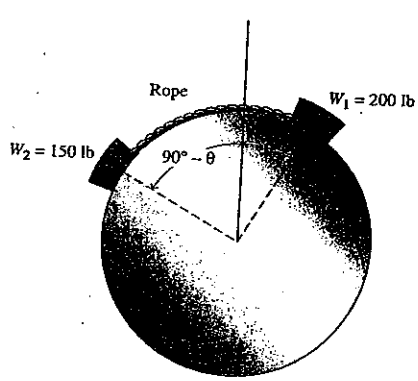


Fig. 4

4. Two bodies weighing 150 lb and 200 lb, respectively, rest on a cylinder and are connected by a rope as shown in Fig. 4. Find the reactions of the cylinder on the bodies, the tension in the rope, and the angle θ. Assume all surfaces to be smooth. (16%)

5. The rope connecting the two blocks of Fig. 5 passes over a fixed drum. The coefficient of friction between the left block and the floor is 0.5; between the right block and the floor, 0.4; and between the rope and the drum, 0.3. Determine the minimum and maximum mass of the left block for which motion does not occur. (18%)

6. A mass (m = 100 kg) and a spring (k =50 kN/m) are attached to a crank, which rotates about a frictionless pin at support O as shown in Fig. 6a. The spring is unstretched when $\theta = 0°$ (θ is defined in Fig. 6b). Determine the angle θ for equilibrium and show that the equilibrium is stable. (16%)

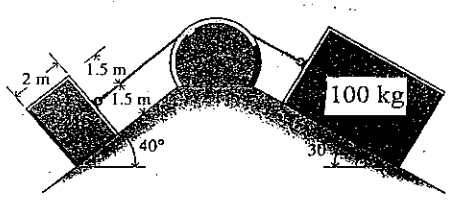
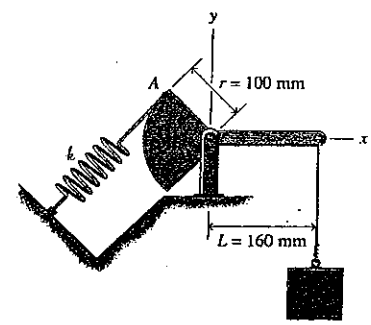
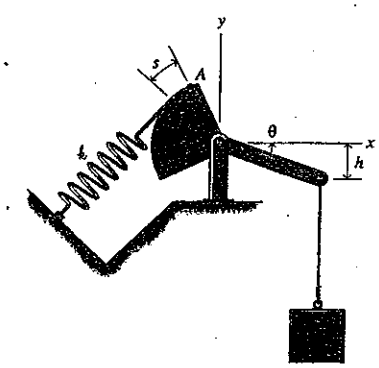


Fig. 5



(a)



(b)

Fig. 6