國立中山大學八十七學年度碩博士班招生考試試題

科　目：工程數學（機械所甲、乙、丙、丁組）

1. (15%) Find the general solution of equation

$$y'' - 2y' + y = e^{x}$$

2. (20%) Solve

$$k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 < x < L,\ t > 0$$

Subject to

$$u(0,t) = 0, \quad u(L,t) = 0, \quad t > 0$$
$$u(x,0) = 10, \quad 0 < x < L$$

3. (10%) (a) Find the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 2 & 1 & 2 \\ 4 & 2 & 4 \\ 2 & 1 & 2 \end{bmatrix}$$

(10%) (b) Is the matrix singular? What is its rank?

4. (10%) Solve the following simultaneous equations by Gaussian elimination

$$2x + 4y - 2z = 2$$
$$4x + 9y - 3z = 8$$
$$-2x - 3y + 7z = 10$$

5. (10%) Find the Laplace transform of the function

$$f(t) = |\sin at|, \quad a > 0$$

6. (10%) Determine the Laurent expansion of the function, $\dfrac{1}{z^2+1}$, about the given point, $z_0 = i$, and determine R so that this expansion is valid in the annulus $0 < |z - z_0| < R$.

7. Let $\mathbf{R} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and $r = |\mathbf{R}| = \sqrt{x^2 + y^2 + z^2}$.

(7%) (a) For what values of n is $\nabla^2 r^n = 0$?

(8%) (b) Determine n so that $\nabla \cdot (r^n \mathbf{R}) = 0$ will vanish identically.

1. Explain the following nomenclatures (20%)
   a. partial property
   b. dead state
   c. Clapeyron equation
   d. flow exergy
   e. van der Waals equation of state
   f. wet-bulb temperature
   g. regenerative cycle
   h. stoichiometric reaction
   i. partial pressure
   j. Joule-Thomson coefficient

2. The energies occured in thermodynamis are always classified into five types of forms: internal energy, heat, work, kinetic energy, and potential energy. Can you tell me what can be stored within medium, and what cannot be stored within medium and are belong to transient energies? (5%)

3. Why does the behavior of a real gas deviate from the behavior of an ideal gas, generally? (5%)

4. Give the explanation of relative humidity, humidity ratio, and wet-bulb temperature? (5%)

5. How do air-standard power cycle (Brayton cycle) differ from vapor power cycle (Rankine cycle)? Why do we usually get low efficiencies for a Brayton cycles than those for Rankine cycles? (5%)

6. The van der Waals equation of state has the form (10%)

$$p = \frac{\overline{R}T}{\overline{v} - b} - \frac{a}{\overline{v}^2}$$

(a) using $\left(\frac{\partial^2 p}{\partial \overline{v}^2}\right)_T = 0$ and $\left(\frac{\partial p}{\partial \overline{v}}\right)_T = 0$ at critical point, show that

$$a = \frac{27}{64}\frac{\overline{R}^2 T_c^2}{p_c} \qquad b = \frac{\overline{R}T_c}{8p_c}$$

(b) Express the equation in terms of the compressibility factor Z, the reduced temperature T_R, and the pseudoreduced specific volume, $\overline{v}'_R = \frac{p_c \overline{v}}{\overline{R}T_c}$.

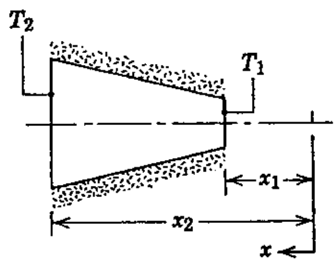
(7) The temperature distribution across a wall 1 m thick at a certain instant of time is given as

$$T(x) = a + bx + cx^2$$

where T is in degrees Celsius and x is in meters, while $a = 900°C$, $b = -300°C/m$, and $c = -50°C/m^2$. A uniform heat generation, $\dot{q} = 1000$ W/m^3, is present in the wall of area 10 m^2 having the properties $\rho = 1600$ kg/m^3, $k = 40$ W/m · K, and $c_p = 4$ kJ/kg · K. (20%)

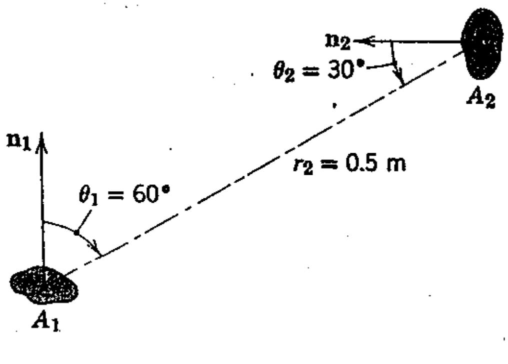
1. Determine the rate of heat transfer entering the wall ($x = 0$) and leaving the wall ($x = 1$ m).
2. Determine the rate of change of energy storage in the wall.
3. Determine the time rate of temperature change at $x = 0$, 0.25, and 0.5 m.

(8) The diagram shows a conical section fabricated from pyroceram. It is of circular cross section with the diameter $D = ax$, where $a = 0.25$. The small end is at $x_1 = 50$ mm and the large end at $x_2 = 250$ mm. The end temperatures are $T_1 = 400$ K and $T_2 = 600$ K, while the lateral surface is well insulated. (15%)



1. Derive an expression for the temperature distribution $T(x)$ in symbolic form, assuming one-dimensional conditions. Sketch the temperature distribution.
2. Calculate the heat rate q_x through the cone.

(9) Consider a small surface of area $A_1 = 10^{-4}$ m^2, which emits diffusely with a total, hemispherical emissive power of $E_1 = 5 \times 10^4$ W/m^2. At what rate is this emission intercepted by a small surface of area $A_2 = 5 \times 10^{-4}$ m^2, which is oriented as shown.
(15%)



1. (20%)

(a) What is Newtonian fluid? (give the meaning for each symbol, if any, you use to answer the question)

(b) Give major assumptions of the Navier-Stokes equations

(c) Write down the assumptions for the Bernoulli equation

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = const. \qquad \text{throughout the flow field.}$$

(d) Under which circumstances can the equations of motion be formulated in terms of a scalar streamfunction alone(give the most general conditions).

(e) Under which circumstances can the equations of motion be formulated in terms of a velocity potential alone (give the most general conditions).

2. A steady, incompressible, two-dimensional flow has the velocity components

$u=4ax-6by$, $v=2cy$ (a, b, and c are not zeros)

(a) How does c depend on a and b? (4%)

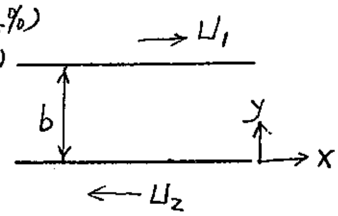
(b) Give the vorticity of the velocity field. Is it rotational or irrotational? (4%)

3. An incompressible, viscous fluid is places between horizontal, infinite, parallel plates as is shown in the figure. The two plates move in opposite directions with velocity, U_1 and U_2 as shown. The pressure gradient in the x direction is zero. Assume laminar, steady flow, and no velocity component in the y-direction.

(a) Write the governing equations for the fluid motion? (4%)

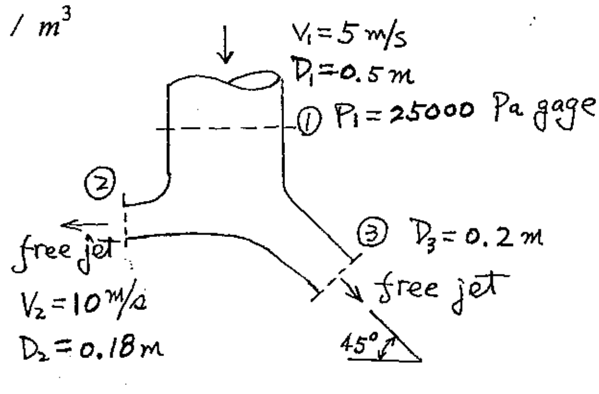
(b) Boundary conditions for the equation of motion? (4%)

(c) Solve for the velocity distribution. (4%)



4. Water is moving steadily through a double-exit elbow for which V_1=5m/s. The inside volume of the elbow is 1 m^3. Find the vertical and horizontal forces from air and water on the elbow. Take V_2=10 m/s, and for water $\rho = 998 kg/m^3$

(10%)



5. The drag on a small, completely submerged solid body having a characteristic length 2.5 mm and moving with a velocity of 15 m/s through water is to be determined with the aid of a model in an unpressurized wind tunnel. The length scale is to be 50, which indicates that the model is to be larger than the prototype. We expect D=f(L, V, ρ, μ), where D=drag, L=characteristic length, V=velocity, ρ=density, and μ=viscosity coefficient. (15%)

(a) what dimensionless parameters would you use to organize these data? i.e. write down the functional relation of the drag in dimensionless form by dimensional analysis

(b) Determine the required velocity in the wind tunnel, V_m=?

(c) Predict the drag on the prototype D, if the drag on the model is measured to be D_m.

Note: For water at 20 ℃, $\mu = 1 \times 10^{-3} kg/m \cdot s$, $\rho = 998 kg/m^3$.

For standard air $\mu = 1.79 \times 10^{-5} kg/m \cdot s$, $\rho = 1.23 kg/m^3$

6. For a uniform flow over a flat plate, plot the local heat transfer coefficient along the plate (i.e. h-x) and discuss the shape of the curve that you have drawn. (10%)

7. For a flow across a circular cylinder, plot the local Nusselt number, Nu_D, along the cylinder surface (i.e. Nu_D-θ) for both Re=$5*10^5$ and $5*10^4$. Discuss the shape of the curve that you have drawn. (10%)

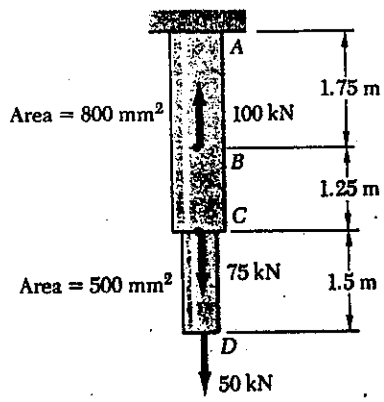
8. Define a thermally fully developed laminar pipe flow. Derive the relation between $\frac{\partial T}{\partial x}$ and $\frac{dT_m}{dx}$ for both constant wall heat flux and constant wall temperature cases. T_m is the bulk mean fluid temperature. What are the values of the Nusselt number, Nu_D, for both cases? (10%)

9. Define each of the following numbers, and state their physical interpretation.(5%)

(1) Prandtl number, Pr

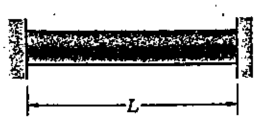
(2) Grashof number, Gr

1. (15%) The rod ABCD is made of an aluminum alloy for which E=70 GPa. For the loading shown, and neglecting the weight of the rod, determine the deflection:(a) of point B, (b) of point D.



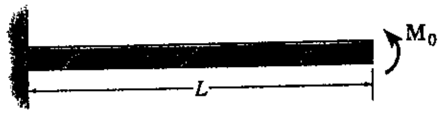
2. (15%) For a plane stress state $\sigma_z = 0$, if the strains ε_x and ε_y have been determined experimentally, determine the expressions of σ_x, σ_y, and ε_z in terms of E, ν, ε_x, and ε_y.

3. (20%) A steel rod of length L and uniform cross section of area A is attached to rigid supports and is unstressed at a temperature of 70°F. The steel is assumed to be elastoplastic with E= 29 x 10^6 psi and σ_y =36 ksi. Knowing that α=6.5 x 10^{-6}/°F, determine (a) the stress in the rod after the temperature has been raised to 300°F, (b) the residual stress after the temperature has returned to 70°F.



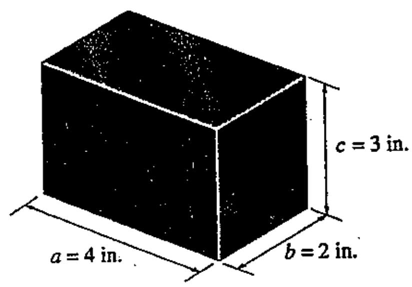
4. Draw the shear and moment diagrams for the beam shown below.

10%



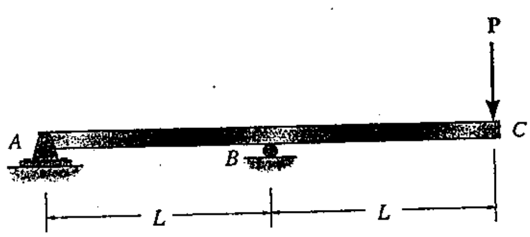
5、 If the rectangular rubber block shown in Fig. below is subjected to a uniform pressure of $p = 20$ psi, determine the dilatation and the change in length of each side. Take $E_r = 600$ psi, $\nu_r = 0.45$.

20%



6、 Determine the bending strain energy in region AB of the beam shown in Fig. below. EI is constant.

20%



Prob.#1 (25%)

A bar and a cable are used to support a cylinder with a mass m =200kg as shown in Fig.1. Determine the internal resisting forces and moment transmitted by section *aa*. Note: The friction forces can be neglected.

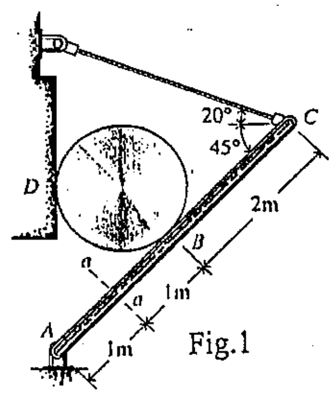


Fig.1

Prob.#2(25%)

The skier Fig.2 descends the smooth slope, which may be approximated by a parabola. If she has a weight of 120 lb, determine the normal force she exerts on the ground at the instant she arrives at point *A*, where her velocity is 30ft/s. Also compute her acceleration at *A*.

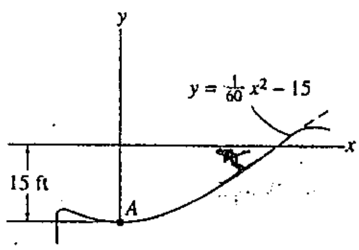


Fig.2

Prob. #3 (25%)

At a given instant the wheel is rotating with the angular velocity and angular acceleration shown. Determine the acceleration of block B at this instant.

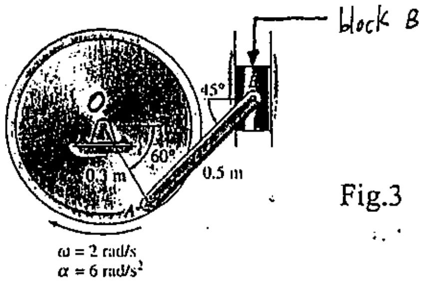


Fig.3

Prob. #4 (25%)

The 20-kg disk is originally at rest and the spring holds it in equilibrium. A couple moment M=30N-m is then applied to the disk as shown. Determine how far s the center of mass of the disk travels down along the incline, measured from the equilibrium position, before it stops. The disk rolls without slipping.

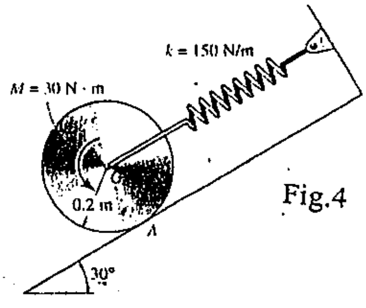


Fig.4

(橫書式)

國立中山大學八十七學年度碩博士班招生考試試題

科　目：自動控制(機械所丙組)　　　　共 1 頁 第 1 頁

1. Given a state-space representation for the system of Figure 1, x_1, x_2, and x_3 are state variables, find
   (a) state equations and output equation in vector-matrix form. (8%)
   (b) transfer function $Y(s)/R(s)$, where $Y(s)$, and $R(s)$ are the Laplace transform of $y(t)$, and $r(t)$, respectively. (10%)
   (c) the range of gain, K, that will cause the system to be stable, unstable, and marginally stable. Assume $K > 0$. (12%)

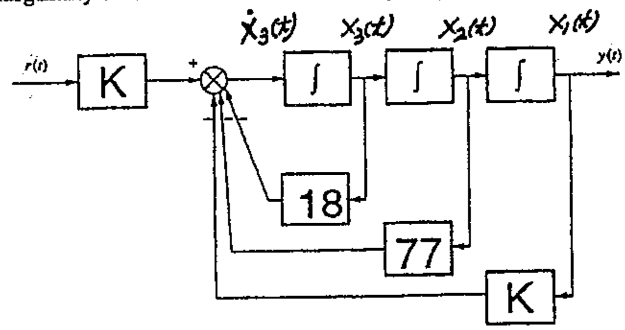


Figure 1

2. For the system shown in Figure 2a, and its equivalent diagram is shown in Figure 2b, the torque T is input, and the angular displacement θ_1 is output. Find the gear ratio, N_1/N_2, so that the settling time, T_s, for a step torque input is 10 seconds. The settling time $T_s = 4/(\zeta\,\omega_n)$ is supposed to be used here, where ζ is damping ratio, ω_n is natural frequency. (20%)

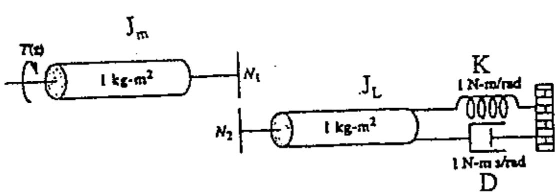


Figure 2a

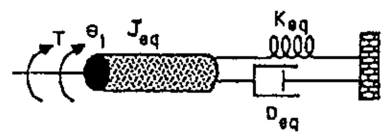


Figure 2b

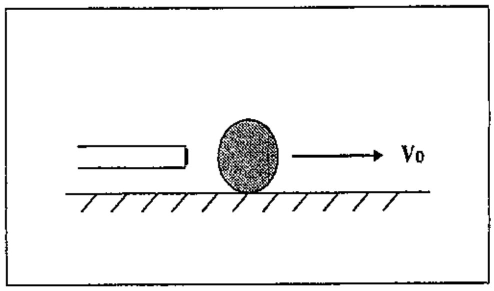
3. The transfer function of a system is G(s), what is the meaning and significance of $|G(j\omega)|$, $\angle G(j\omega)$?
   If the input signal is sin(ωt), how about the output ? (15%)

4. What is the difference between root-locus design approach and frequency domain design approach? (20%)

5. What is the definition and significance of gain margin and phase margin? How to get them through Bode plot? (15%)

1. (25%)

A billiard ball is struck by a cure as in the given figure. The line of action of the applied impulse is horizontal and passes through the center of the ball. The initial velocity V_o of the ball after the impact, its radius R, its mass M, and the coefficient of friction μ between the ball and the table are all known. How far will the ball move before it ceases to slip on the table and starts to roll.

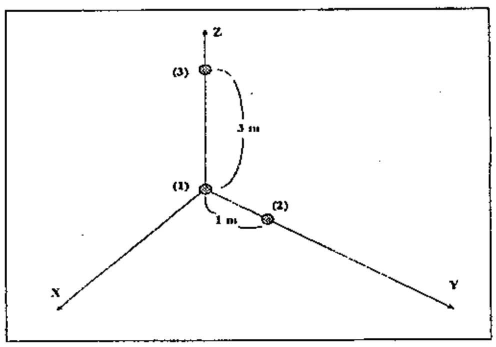


2. (25%)

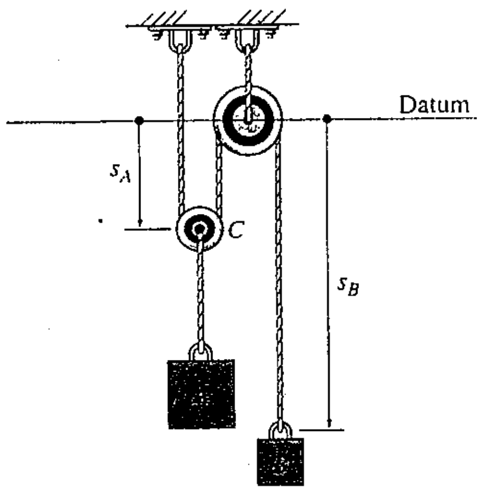
Three charged particles in a vacuum are shown in the given figure. Particle 1 has a mass of 10^{-5} kg and a charge of 4×10^{-5} C (coulombs) and is at the origin at the instant of interest. Particles 2 and 3 each have a mass of 2×10^{-5} kg and a charge of 5×10^{-5} C and are located respectively at the instant of interest 1 m along the y axis and 3 m along the z axis. An electric field $\boldsymbol{E}$ given as

$$E = 2xi + 3zj + 3(y + z^2)k \ \ N/C$$

is imposed from the outside. Compute (a) the position of the center of mass for the system, (b) the acceleration of the center of mass. Note that two external forces act on each particle: the force of gravity and the electrostatic force form the external field. Recall from physics that this electrostatic force is given as qE, where q is the charge on the particle.

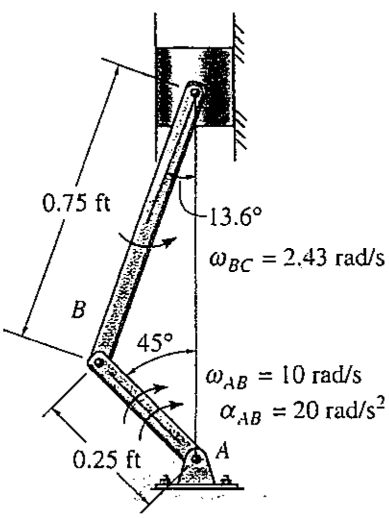


3. (25%)



The 100-kg block A is released from rest. If the mass of the pulleys and the cord is neglected, determine the speed of the 20-kg block B in 2 seconds.

4. (25%)



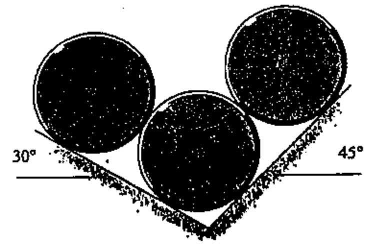
The crankshaft AB turns with a clockwise angular acceleration of 20 rad/s^2. Determine the acceleration of the piston at the instant AB is in the position shown. At this instant $\omega_{AB} = 10$ rad/s and $\omega_{BC} = 2.43$ rad/s.

（橫書式）

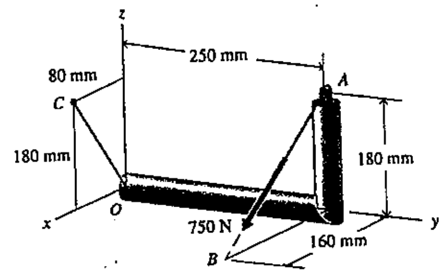
國立中山大學八十七學年度碩博士班招生考試試題

科　目：靜力學（機械所丁組）　　　　共 1 頁　第 1 頁

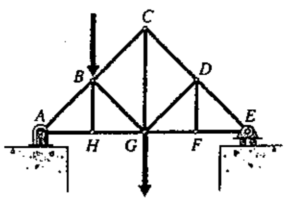
1. Three homogeneous cylinders A, B, and C are stacked in a V-shaped trough. Each cylinder has a diameter of 500 mm and a mass of 100 kg. Determine the forces exerted on cylinder A by the inclined surfaces. Assume that all surfaces are smooth. (15%)



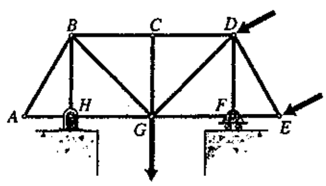
2. Determine the scalar component of the moment of the 750-N force about line OC.(15%)



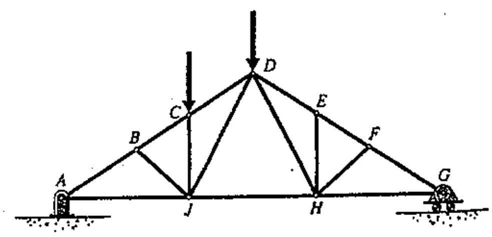
3. Identify the zero-force members present when the trusses shown in the following problems are subjected to the loading indicated. (10%)



a

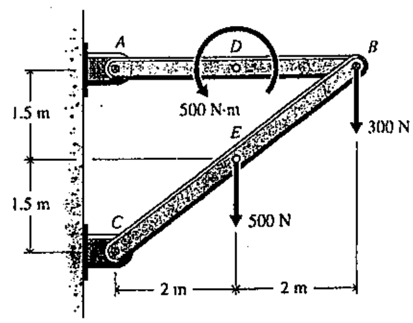


b

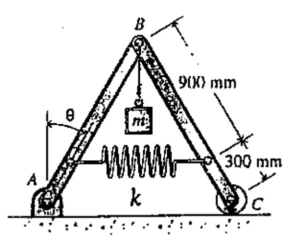


c

4. Determine the horizontal and vertical components of the reaction at support A of the two-bar frame.(20%)



5. A two-bar mechanism supports a body (m = 75 kg). The unstretched length of the spring is 600 mm. Determine the spring stiffness k required to limit the angle θ for equilibrium to 35 degree. Assume that the masses of the bars are negligible.(20%)



6. The rods are lightweight and all pins are frictionless. The coefficient of friction between the 40-kg slider block and the floor is 0.40. Determine
   a. The maximum force P for which motion does not occur if the force P is horizontal (θ = 0).
   b. The angle θ that gives the absolute greatest forces P for which motion does not occur.　　　(20%)